NATIONAL MATHEMATICS MAGAZINE

(Formerly Mathematics News Letter)

Vol. XI BATON ROUGE, LA., OCTOBER, 1936 No. 1

PUBLISHED BY LOUISIANA STATE UNIVERSITY

Every paper on technical mathematics offered for publication should be submitted with return postage to the Chairman of the appropriate Committee, or to a Committee member whom the Chairman may designate to examine it, after being requested to do so by the writer. If approved for publication, the Committee will forward it to the Editor and Manager at Baton Rouge, who will notify the writer of its acceptance for publication. If the paper is not approved the Committee will so notify the Editor and Manager, who will inform the writer accordingly.

1. All manuscripts should be typewritten, with double spacing and with margins at least one inch wide.

2. The name of the Chairman of each committee is the first in the list of the committee.

3. All manuscripts should be worded exactly as the author wishes them to appear in the Magazine.

Papers intended for the Teacher's Department, Department of History of Mathematics, Book Reviews or Problem Department should be sent to the respective Chairmen.

Committee on Algebra and Number Theory: L. E. Bush, W. Vann Parker, James McGiffert.

Committee on Analysis and Geometry: W. E. Byrne, Wilson L. Miser, Dorothy McCoy, H. L. Smith.

Committee on Teaching of Mathematics: Joseph Seidlin.

Committee on Statistics: C. D. Smith, Irby C. Nichols.

Committee on Mathematical Notes: L. J. Adams.

Committee on Book Reviews: P. K. Smith.

Committee on Problem Department: T. A. Bickerstaff.

Committee on Humanism and History of Mathematics—G. Waldo Dunnington.

Published by the Louisiana State University Press

th lib
ch.
isiana state univ.

Club Rates
May Be Had
on Application

Subscription, $1.50
Per Year in Advance
Single Copies, 20c.

Vol. XI BATON ROUGE, LA., OCTOBER, 1936 No. 1

Published 8 Times Each Year by Louisiana State University. Vols. 1-8 Published as Mathematics News Letter.

All Business Communications should be addressed to the Editor and Manager P. O. Box 1322, Baton Rouge, La.

This Journal is dedicated to the following aims:

1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.
2. To supply an additional medium for the publication of expository mathematical articles.
3. To promote more scientific methods of teaching mathematics.
4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Professor Seidlin's Questionnaire on Mathematics Teaching

The cream of the replies returned to Editor Seidlin's questionnaire (see below) is found in the Teacher's Department of this issue of the Magazine. A "general tabulation" of all replies returned to him (nearly 800 of the 1000 mailed out) was published in our May issue. It will be well for the readers who desire to arrive at the most intelligent interpretation of the total returns to this significant body of questions to read again the Teacher's Department material of the May issue, Vol. 10, of National Mathematics Magazine.

For the greater convenience of our readers of this the second installment of the Seidlin story, the entire questionnaire is once more published. Thus is it made easy for them to check with the original questions all references to the same as found in the replies offered:

I. *Do you believe that at least one year of algebra and one year of geometry should be required of all high school pupils*:

(*a*) *As now taught?* . . . . [ ]

(*b*) *Changed* (*presumably improved*) *in content?* . . . . [ ]

II. *Do you believe that at least six semester hours of mathematics should be required of all students in a liberal arts college?* . . . . [ ]

III. *Do you believe that mathematics at the level indicated below, requires "special mathematical ability?"*

(*a*) *Ninth grade* . . . . [ ]

(*b*) *Tenth grade* . . . . [ ]

(*c*) *First year college* . . . . [ ]

IV. *Do you believe that only "superior" or "scientifically-minded" students should be encouraged to elect mathematics beyond the 8th, 9th, 10th, 11th, 12th, 13th* (*first year college*) *grade? Indicate grade.* . . . . [ ]

V. *Comments*:

S. T. SANDERS.

On Measures and Weights by Epiphanius

By ALLEN A. SHAW
University of Arizona

St. Epiphanius (315-403) was a famous Eastern Church Father, a native of Bezanduke near Eleutheropolis in Judaea, and a monk from his earliest youth. He was an intimate friend of St. Hilarion, a celebrated Palestinian monk, and later of St. Jerome who called him Pentaglossos (Five-tongued) in allusion to his knowledge of Hebrew, Syriac, Egyptian, Greek and Latin. He visited Egypt in search of more advanced studies. When about twenty-five years of age, he returned to his native land, and founded a monastery near Eleutheropolis over which he presided for some thirty years, in the course of which time he was ordained to the priesthood. The fame of his piety and great learning induced the bishop of Cyprus to choose him in 367 as their bishop of Salamis (Constantia). In this capacity he became celebrated for his holy life, his spread of monasticism, and especially his fiery defence of orthodoxy. He was a strong opponent of Origenism, both in his writings and in his discourses. Indeed, he preached and wrote unceasingly against all sorts of heresies, heresies of his own and the preceding centuries. He died at sea at an advanced age, A. D. 403.

His writings are polemical and biblo-archaeological. To the former belong his ancoratus[1] i. e. (the firmly-anchored man). Its purpose is to afford a solid anchorage to those of the faithful who are cast about on the waves of heretical conflicts. This work ends with two Professions of Faith, of which, according to Caspari, Epiphanius was the author of the longer one. Later on, by requests of friends, he wrote his greatest treatise, Panarion[2] or medicine-chest against eighty heresies. This work proposes to furnish an antidote to those who have been bitten by the serpent of heresy, and to protect those who are orthodox.

Of the biblo-archaeological writings his most important work is *Measures and Weights*[3] which he composed in Constantinople in 392 for a Persian priest. This work contains much more than the title indicates, and consists of three parts: the first part treats of the canon and versions of the Old Testament, the second describes measures

[1] ἀγκυρωτος.
[2] πανάριου or πανάρια.
[3] περὶ μέτρω καὶ σταθμῶν.

and weights, and the third treats of the geography of Palestine. The work seems more like a collection of notes and sketches than a finished treatise as we shall see presently. It is, however, very important for the metrologist and the historian, as it influenced future metrologists to compose more systematic treatises on the subject. For example, Ananiah Shiragooni, the distinguished Armenian astronomer and mathematician (7th cent.), was greatly indebted to Epiphanius in the composition of his *Weights and Measures* frequently referred to in the present writer's commentary of Epiphanius. The corresponding work of Shiragooni is shorter and clearer and more systematic. In fact, all the obscure or doubtful passages of Epiphanius are elucidated by Shiragooni. Besides, in Shiragooni, each measure of weight, e. g. uncia, litra, mnasis, talent, etc. is expressed in terms of lower units and finally in barley-grains.

The metrology proper of Epiphanius describes the following weights and measures in the order as they are listed in paragraph 3, p. 2:

Lethec, Gomor, Batus, Saton, Modius, Cabus, Choenix, Hype of fine flour, Drax of flour, Artabe, Three Measures of fine flour, Three Cana of groats, Nebel of wine, Collathon, Alabastrum of sweetoil, Campsakis of water, Cotyla of oil, Cyathus, Metretes of wine, Metretes of oil, Tryblion, Xestes, Amphoreus, Aporryma, Sabitha, Hin, Chous, golden Stamnus, where manna was kept, Mares, Cyprus, Congiarion.

Epiphanius takes this list and explains each measure in order up to Chous inclusive. He omits the explanation of golden Stamnus[4] and suddenly breaks the continuity of the subject of measures of capacity and starts (p. 15) with the explanation of the following measures of weight:

Talentum, Lepton, Assarion, Denarius, Argyrus, Litra, Uncia, Stater, Didrachma, Shekel or Codrantes, Drachma, Obolus, Chalcus or Chalcina, Mnas, Numion, Dichryson, Meliarisium, Pholes or Talantion.

After his description of this list, Epiphanius resumes his explanation of the remaining three measures of his first list, namely, Mares, Cyprus and Congiarion. The second part of the treatise ends with the description of Congiarion. This lack of order in his descriptions, omission of the explanation of "golden Stamnus," a few inconsistencies in his definitions, presence of irrelevant passages (pp. 5-11) as pointed out by Petavius, etc. indicate that his work was a collection of notes and sketches rather than a finished composition.

[4] See Ex. 16:33 and Heb. 9:4.

Consider now any one of the measures, say, Talentum, as an illustration:

Epiphanius says: "Talentum is the greatest of all the measures of weight, and in litra weight, it is 125 litrae."

Shiragooni states: "Talentum is 125 litrae. Metrologists (lit. computers) call this Monados, and, expressing it in terms of lower units, we have

1 talentum =	1500 unciae
=	6000 shekels
=	12000 staters
=	18000 semisses
=	27000 trimesses
=	36000 grammaria
=	54000 sinks
=	216000 ceratia
=	288000 pshid, . . . (= obolus)
=	432000 danks (= assarion)
=	864000 barley-grains."

The reader can see how much fuller and nore analytic is the description of Shiragooni. Throughout the commentary of Epiphanius[5] the corresponding passages of Shiragooni are quoted and then commented on the term in question.

Testimonies of other classical writers, Greek Latin and Armenian, confirm the equivalence of talentum and 125 litrae in the time of Epiphanius and Shiragooni. For example, John Yerzengatzi, an Armenian authority of the twelfth century, in this commentary of St. Matthew (18:24) says: "Talentum is kankar (Heb. Kikkar) and is equal to 125 litra, and in weight is 9000 tahegan." And again in his exposition of Matt. 25:15 he adds: "Talent is kankar in Hebrew, which is 9000 tahegan."

The question which naturally arises here is this: what talentum is meant here by Epiphanius or Shiragooni? For the ancients used various kinds of talents—the Babylonian (32.64 kg, or 40.8 kg), the Egyptian (42.5 kg), the Assyrian (29.376 kg), the Ptolemaic (21.25 kg), etc., all different in weights and subject to modifications at different times and places. Though these talents were all different in weight, yet there is a simple arithmetical relation between them, constantly pointed out by ancient metrologists, especially by Alexandrine Greeks, and we have the following relation: 1 Assyrian talent = 9/10 of Baby-

[5] See May issue (1936) of the *Bulletin* of the American Mathematical Society.

lonian talent $=24/25$ of Egyptian talent $=4/5$ of Babylonian canthar $=12/25$ of Ptolemaic talent.

Now expert metrologists agree that Babylonia was the ancient home of all scientific metrology, and that the most ancient systems of weights and measures were of Babylonian origin. Since their sexagesimal system was thoroughly scientific, their weights and measures were constructed with rigid scientific accuracy upon the basis, astronomically ascertained, of the unit of length the cube of which gave the limit for measures of capacity, and the weight of water contained in this unit-cube constituted the unit of weight[6]. But we must remember that in the constituion of talents and other meausres of weights, it is by starting with the *weight* that the dimension or length of foot is determined. For, with the ancients the idea of *weight* seems to have rapidly predominated over that of volume, the measure of capacity: Every commercial product was sold in weights: gold, barley, oil, honey, wheat, flour, etc. In order to avoid trouble of measuring these products, the ancients *weighed* them. To this end they assigned to these products a *conventional* density. They fixed

for barley and honey 2/3 of the weight of water,
for wheat 4/5 of the weight of water,
for oil 9/10 of the weight of water,
for flour of wheat 25/27 of the weight of water,
for flour of barley 3/4 of the weight of water, etc.

Decourdemanche (p. 2ff and p. 22) has shown that this talent was the Babylonian canthar of 40.8 kg, which if divided by 125 litrae (the Roman-Byzantine libra) gives 326.4 gm for a litra. We have adopted this value of litra as our fundamental unit to be used in the calculation of the remaining measures of weights. It is to be noted here that the weights of librae in the Louvre Museum (No. 532 and 533) are 324gm, and the difference with the above result is very small. We have given preference to the theoretical value of Decourdemanche as it makes our calculations easier.

Though not relevant to the subject, it is very important to note that the cube roots of 42.5kg, 21.25kg, 32.64kg and 29.376kg give the Egyptian, Ptolemaic, Babylonian and Assyrian *foot* respectively, which are, in the above order, 349mm (13.74ins), 277mm (11.56ins), 319.6mm (12.582ins) and 308.56mm (12.15ins). For example, cube root of

[6] See Decourdemanche, J. A. Traité pratique des poids et mesures des peuples anciens et des arabes, Paris, 1909. P. 2ff., also Kenedy, A. R. S.: *Weights and Measures* in *A Dictionary of the Bible*, Hastings,—a very complete and satisfactory article.

29.376kg = cube root of 29.376 liter equivalence in water of 29.376kg = 308.56mm, = the length of the Assyrian foot = Greek foot. The equivalence of Assyrian and Greek foot is due to the fact that the exterior columns of the Parthenon in Athens are $33^1/_3$ Greek feet or $10.285^1/_3$m in height. From this we readily get one Greek foot equals 308.56mm. This agreement between an architectural measure and a monetary unit (17gm = 1 Attic tetradrachma) has led to the adoption of 17/4 grams for one drachma, which can also be used as a fundamental unit in our calculations, e. g. an Egyptian talent of 10,000 drachmae $= (17/4) \times 10{,}000 = 42.5$kg. A Babylonian canthar of 9600 drachmae $= (17/4) \times 9600 = 40.8$kg. A Babylonian talent of 7680 drachmae $= (17/4) \times 7680 = 32.64$kg., etc.

Petavius, the classical authority on Epiphanius, in his commentary of talentum (Opera Omnia, Paris, 1622) says: "For talentum and its varieties, we have innumerable statements by commentators. By talentum here Epiphanius means the Jewish talent of Holiness which is 120 mnas, and, according to the Romans, 125 litrae, for a litra is 96 drachmae, mnas 100 (drachmae), 120 mnas contains 12000 drachmae which divided by 96 gives us the number 125 litra according to the Romans." Our exposition above indicates that Petavius is incorrect here. We must take the talent of Epiphanius as a talent of 9600 drachmae and not of 12000 which makes the talent 51kg.

The remaining weights and measures are in like manner critically explained in the commentary of our translation of Epiphanius. The translation gives variant readings from the text of P. de Lagarde or doubtful passages. The work includes a Greek-English vocabulary of the text, the Greek text itself and a bibliography. Since frequent reference is made in this work to the Weights and Measures of Ananiah Shiragooni, some account is also given of the latter in the form of an appendix. In fact, all parallel passages from Shiragooni are quoted in the commentary as illustration or explanation. The translation is made from the Venice edition (1821) published by the Mukhtharist Order of St. Lazar, Venice, but the editor has compared this text with that of P. de Lagarde's Symmicta II (1880), and all the textual differences are pointed out in the notes. As remarked above, for purposes of calculation we have found it convenient to adopt the Roman-Byzantine litra (326.4gm) and the Roman Xestes (408gm) as our fundamental units.

A Generalization of Chevilliet's Formula*

By CHESTER C. CAMP
University of Nebraska

There are various methods of approach to the problem of finding further terms in the expansion of the remainder in Simpson's Rule beyond that given by Chévilliet, i. e. terms involving higher derivatives at both ends of the interval. These assume practical importance when the third derivative has the same value at both ends as in the case of

$$\int_0^1 \frac{dx}{(1+x^2)}.$$

The empirical method here leads to the fact that the error varies directly as h^6 if one tries successively $h=1/10$, $1/20$, $1/40$ and finds the ratio of the differences to be 63.95. The true value may now be estimated by adding $1/(64-1)$ of the smallest difference to the best value obtained by Simpson's Rule. In this case the result is correct to 16 decimal places when ordinates are calculated that far. The reliability of such an extrapolation may be tested by the constancy of further ratios of differences when h is successively divided by 2.

Another approach is as follows. Assume

$$\int_a^b f(x)dx = I_{1/3} - h^4[f'''(b) - f'''(a)]/180 \\ + Ah^5[f^{IV}(b) - f^{IV}(a)] + Bh^6[f^{V}(b) - f^{V}(a)] + \dots$$

where $I_{1/3}$ represents Simpson's formula and A, B, etc. are to be determined. By taking $a=-1$, $b=1$, $h=1$, $f(x)=x^5$ one readily finds that $A=0$. Take in succession $f(x)=x^6, x^7, x^8, x^9, \dots$ and the coefficients may be determined by this recurrence process: $B=1/1512$, $C=0$, $D=-1/14400$, etc.

A much more complicated procedure was used by Uspensky† in determining the terms not only for the remainder in Simpson's Rule but for other Newton-Cotes formulas. It is the purpose of the present

*Presented to the American Mathematical Society, Nov. 30, 1934.

†On the Expansion of the Remainder in the Newton-Cotes Formula, Transactions of the American Mathematical Society, vol. 37, No. 3, pp. 381-396, May, 1935.

paper to obtain expansions in several important cases of the Newton-Cotes formula by a much simpler method.

The Euler-Maclaurin formula may be written

(1) $$f(a+h)+f(a+2h)+\ldots+f(b-2h)+f(b-h)=$$

$$\frac{1}{h}\int_a^b f(x)dx-\frac{f(b)+f(a)}{2}+\frac{B_1h}{2!}[f'(b)-f'(a)]$$

$$-\frac{B_2h^3}{4!}[f'''(b)-f'''(a)]+\ldots+\frac{(-1)^{r+1}B_rh^{2r}s}{(2r)!}f^{(2r)}(a+\theta sh)$$

where $B_1=1/6$, $B_2=1/30$, $B_3=1/42$, etc., $b-a=sh$, $0<\theta<1$. Likewise when $s=2s'$

(2) $$f(a+2h)+f(a+4h)+\ldots+f(b-2h)=$$

$$\frac{1}{2h}\int_a^b f(x)dx-\frac{f(b)+f(a)}{2}+\frac{B_12h}{2!}[f'(b)-f'(a)]$$

$$-\frac{B_22^3h^3}{4!}[f'''(b)-f'''(a)]+\ldots+(-1)^{r+1}\frac{B_r(2h)^{2r}s'}{(2r)!}\times$$

$$f^{(2r)}(a+\theta'sh), 0<\theta'<1.$$

Take (1) $\times 4h/3$ minus (2) $\times 2h/3$ plus $h/3[f(b)+f(a)]$ and obtain for Simpson's Rule $h/3[f(a)+4f(a+h)+2f(a+2h)+\ldots+2f(b-2h)+4f(b-h)+f(b)]$,

(3) $$I_{1/3}=\int_a^b f(x)dx+\frac{(2^4-4)}{3}\frac{B_2h^4}{4!}[f'''(b)-f'''(a)]$$

$$-\frac{(2^6-4)}{3}\frac{B_3h^6}{6!}[f^V(b)-f^V(a)]+\ldots$$

$$+(-1)^{r-1}\frac{(2^{2r-2}-4)}{3}\frac{B_{r-1}h^{2r-2}}{2r-2!}[f^{(2r-3)}(b)-f^{(2r-3)}(a)]+R_r$$

where $$|R_r|<\frac{(2^{2r}+4)}{3}\frac{B_rh^{2r+1}s}{(2r)!}M \text{ and } M\geqq|f^{(2r)}(x)|,\ a\leqq x\leqq b.$$

In case all the even derivatives of $f(x)$ of order $\geqq 2r$ are of the same sign, $a \leqq x \leqq b$, then $|R_r|$ will be

$$\leqq \frac{B_r(2^{2r}+4)}{3(2r)!} h^{2r}[f^{(2r-1)}(b)-f^{(2r-1)}(a)],$$

which for large r is practically the size of the next term.

A similar treatment might be given for the cases of the Newton-Cotes formula when $n=3,4,6$, as well as for Weddle's and Hardy's rules. For further simplicity and for easy comparison with Uspensky's results it is now expedient to consider the choice $a=0$, $b=1$, $h=1/n$, and to confine attention to the general term of the expanded remainder. The form of R_r may be written down by analogy from the discussion above.

Define

$$(4)\quad \varphi(n) \equiv f(1/n)+f(2/n)+f(3/n)+\ldots+f(1-1/n) = (n-1)\int_0^1 f(x)dx$$

$$+\frac{B_1}{2!}(-1+1/n)[f'(1)-f'(0)] - \frac{B_2}{4!}(-1+1/n^3)[f^{III}(1)-f^{III}(0)]$$

$$+\frac{B_3}{6!}(-1+1/n^5)\times[f^V(1)-f^V(0)]-\ldots$$

$$+\frac{(-1)^{k-1}B_k}{(2k)!}(-1+1/n^{2k-1})[f^{(2k-1)}(1)-f^{(2k-1)}(0)]+\ldots$$

Consider also the simpler case of the Euler-Maclaurin formula

$$(5)\quad \tfrac{1}{2}f(0)+\tfrac{1}{2}f(1) = \int_0^1 f(x)dx + B_1[f'(1)-f'(0)]/2!$$

$$-B_2[f'''(1)-f'''(0)]/4! + B_3[f^V(1)-f^V(0)]/6! - \ldots$$

$$+(-1)^{k-1}B_k[f^{(2k-1)}(1)-f^{(2k-1)}(0)]/(2k)!+\ldots \equiv \varphi.$$

For Newton's rule take $3\varphi(3)/8+\tfrac{1}{4}\varphi$, obtaining

$$[f(0)+3f(1/3)+3f(2/3+f(1)]/8 = \int_0^1 f(x)dx$$

$$-B_2(-1+1/3^2)[f^{III}(1)-f^{III}(0)]/4!8+\frac{B_3}{6!8}(-1+1/3^4)[f^V(1)-f^V(0)]$$

$$-\ldots+(-1)^{k-1}B_k(-1+1/3^{2k-2})\left[\frac{f^{2k-1}(1)-f^{2k-1}(0)}{(2k)!8}\right]+\ldots$$

For Boole's rule take $16\varphi(4)/45-10\varphi(2)/45+7\varphi/45$ and get

$$[7f(0)+32f(\tfrac{1}{4})+12f(\tfrac{1}{2})+32f(\tfrac{3}{4})+7f(1)]/90=\int_0^1 f(x)dx$$

$$+B_3[f^V(1)-f^V(0)]/6!64+\ldots+(-1)^{k-1}B_k(1+1/4^{2k-3}$$
$$-5/2^{(2k-2)})[f^{(2k-1)}(1)-f^{(2k-1)}(0)]/(2k)!45+\ldots.$$

For Weddle's rule one uses $[2\varphi+5\varphi(6)-4\varphi(3)+\varphi(2)]/20$, getting

$$[f(0)+5f(1/6+f(1/3)+6f(\tfrac{1}{2})+f(2/3)+5f(5/6)+f(1)]/20$$

$$=\int_0^1 f(x)dx+\frac{B_3}{6!20}(1/2^5-4/3^5+5/6^5)[f^V(1)-f^V(0)]+\ldots.$$

$$+\frac{(-1)^{k-1}B_k}{(2k)!20}\left(\frac{1}{2^{2k-1}}-\frac{4}{3^{2k-1}}+\frac{5}{6^{2k-1}}\right)\times[f^{(2k-1)}(1)-f^{(2k-1)}(0)]+\ldots$$

For Cotes' rule where $n=6$ take $82\varphi+216\varphi(6)-189\varphi(3)+56\varphi(2)$ and divide by 840, obtaining

$$[41f(0)+216f(1/6)+27f(1/3)+272f(\tfrac{1}{2})+27f(2/3)+216f(5/6)$$

$$+41f(1)]/840=\int_0^1 f(x)dx\quad+\frac{B^4}{8!6^4}[f^{VII}(1)-f^{VII}(0)]$$

$$+\sum_{k=5}\frac{(-1)^{k-1}B_k}{(2k)!840}\left(\frac{1}{6^{2k-4}}-\frac{7}{3^{2k-4}}+\frac{7}{2^{2k-4}}-1\right)\times[f^{(2k-1)}(1)-f^{(2k-1)}(0)].$$

For Hardy's rule take $28\varphi+81\varphi(6)-81\varphi(3)+29\varphi(2)$ and divide by 300, getting

$$[14f(0)+81f(1/6)+110f(\tfrac{1}{2})+81f(5/6)+14f(1)]/300$$

$$=\int_0^1 f(x)dx-\frac{B_3}{6!720}[f^V(1)-f^V(0)]+\ldots$$

$$+\frac{(-1)^{k-1}B_k}{(2k)!300}\left[\frac{81}{6^{2k-1}}-\frac{81}{3^{2k-1}}+\frac{29}{2^{2k-1}}-1\right]\times[f^{(2k-1)}(1)-f^{(2k-1)}(0)]+\dots$$

For Cotes' rule $n=5$ one may obtain coefficients by assuming

$$[19f(0)+75f(1/5)+50f(2/5)+50f(3/5)+75f(4/5)+19f(1)]/288$$
$$=\int_0^1 f(x)dx+A_1[f'(1)-f'(0)]+\sum_{i=2}A_i[f^{(i)}(1)-f^{(i)}(0)]$$

and substituting in succession $f(x)=x^2,x^3$, etc. This has the disadvantage of not giving a general form for A_i.

Another method is to expand $f(k-s)+f(k+s)$ about $x=k$ and then use the alternate form of the Euler-Maclaurin series to convert the several terms into expressions involving the odd ordered derivatives. Thus one may use the formula

$$(6)\qquad f(k)=1/2h\int_a^b f(x)dx-\frac{B_1h}{2!}[f'(b)-f'(a)]$$
$$+\frac{B_2(2^3-1)h^3}{4!}[f'''(b)-f'''(a)]-\dots$$
$$+\frac{(-1)^rB_r(2^{2r-1}-1)h^{2r-1}}{(2r)!}[f^{(2r-1)}(b)-f^{(2r-1)}(a)]+\dots.$$

where $k=a+h=b-h$, by putting $a=0$, $b=1$, $k=\frac{1}{2}$. This is also to be used in replacing $f(k)$ by $f''(k)$, $f^{IV}(k)$, etc. This affords an expression for the general coefficient which will not be simple unless one notices the connection with the Bernoulli polynomials.

Probably the simplest method here would be to use the formula given by Steffansen* and group the terms in pairs thus:

$$(7)\qquad f(\tfrac{1}{2}-s)+f(\tfrac{1}{2}+s)=$$
$$2\int_0^1 f(x)dx+2\sum_{r=1}^{m}\frac{B_{2r}(s+\frac{1}{2})}{(2r)!}[f^{(2r-1)}(1)-f^{(2r-1)}(0)]-R$$

where $$R=\frac{1}{(2m)!}\int_0^1[\overline{B}_{2m}(\tfrac{1}{2}-s-t)+\overline{B}_{2m}(\tfrac{1}{2}+s-t)]f^{(2m)}(t)dt,$$

*Interpolation, 1927, p. 131, formula (5) for the case $x=0$.

and $\bar{B}_{2m}(x)$ is the ordinary extended Bernoulli polynomial function. By putting $s=5/10$, $3/10$, $1/10$ in (7) using as multipliers 19, 75, 50, respectively, adding, and dividing by 288 one may get the general term. The result of either of the last two methods will be equivalent to Uspensky's* formula (17), p. 394, for $n=5$. By the first method one may easily work out a recurrence relation for successive coefficients.

The formula (7) has two other important applications. By it if one substitutes the values of s one may determine the Cotes coefficients by assuming undetermined multipliers and expressing the condition that upon combining by addition the coefficients of

$$\int_0^1 f(x)dx,\ f'(1)-f'(0),\ f'''(1)-f'''(0),$$

shall be 1, 0, 0, respectively.

Secondly, new formulas of approximate integration may be derived by solving for one or more values of s when the multipliers are given. This is done by making the combined coefficient of $\int_0^1 f(x)dx$ unity and those of $f'(1)-f'(0)$, $f'''(1)-f'''(0)$, etc., zero as far as may be. One of the simplest cases of this is the formula used by engineers in measuring velocities in streams:

$$\int_0^1 f(x)dx=\tfrac{1}{2}[f(\tfrac{1}{2}-\sqrt{3}/6)+f(\tfrac{1}{2}+\sqrt{3}/6)],\text{ i. e. } s=\sqrt{3}/6.$$

Obviously (7) may also be employed to test the accuracy of other approximate integration formulas, such as those of Shovelton, Woolhouse, and Sheppard.

*Loc. cit.

Humanism and History of Mathematics

Edited by

G. WALDO DUNNINGTON

Biographical Sketch

OTTO NEUGEBAUER

Professor Otto Neugebauer was born May 26, 1899, at Insbruck in Tyrol, the son of a railway construction engineer, Rudolph Neugebauer. His parents died when he was quite young; his boyhood was spent at Graz in Styria, where he grew up and attended the secondary school, graduating in March, 1917. From October 1917 until November, 1918, he was in military service on the field, with an Austrian mountain battery principally on the Italian front. At the signing of the armistice he was taken prisoner by the Italians. Returning in the fall of 1919, he studied mathematical physics at the University of Graz under Michael Radaković and Roland Weitzenböck. While studying at the University of Munich in 1921-1922 under Arthur Rosenthal and Arnold Johannes Wilhelm Sommerfeld, Neugebauer was so stimulated by their lectures that he decided to devote his life to mathematics. Moving on to Göttingen the following year, he studied mathematics under Professors Richard Courant, Edmund Georg Hermann Landau, and the late Emmy Noether, Egyptian under Professors Hermann Kees and the late Kurt Sethe.

At the University of Göttingen, Neugebauer became an assistant in the department of mathematics in the fall of 1923, the following October (1924) special assistant to Courant, at that time head of the department. Göttingen conferred the Ph.D. on him in 1926; the doctoral thesis is a study of Egyptian fractions. He received the "venia legendi" for the history of mathematics (December 17, 1927) and began lecturing several months later. Further promotion came in 1930 to "Oberassistent" and on January 26, 1932, from Privatdozent to associate professor. Neugebauer was granted at his own request a leave of absence from Göttingen on June 4, 1934, and went to the University of Copenhagen where he has since remained. He is married and has two children. For purposes of scientific research he had previously (spring, 1924) spent some time with Harald Bohr in Copenhagen, with Father Deimel in Rome on Sumerian, and in the fall of 1928 with W. W. Struve and B. A. Turajeff in Leningrad.

As part of his manifold activities Prof. Neugebauer edits two important periodicals "*Zentralblatt für Mathematik und ihre Grenzgebiete*" and the "*Zentralblatt für Mechanik*"; in addition, he edits the two valuable series of monographs, the "*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*" and the "*Ergebnisse der Mathematik und ihrer Grenzgebiete*". In 1932 appeared no less than six distinct contributions from his pen dealing with the history of ancient algebra, the sexagesimal system and Babylonian fractions, Apollonius, Babylonain series, square root approximations, and siege calculations. A later paper covers formulas for the volume of a truncated pyramid in pre-Grecian mathematics (1933), followed by monographs on the origin of the sexagesimal system, the geometry of the circle,and the application of astronomy to chronology in Babylon.

Professor Neugebauer has announced a series of three volumes on the history of ancient astronomy and mathematics. The first volume is "*Pre-Grecian Mathematics*" (1934). Volume two will treat "Grecian Mathematics" and volume three "Babylonian and Grecian Astronomy". The author has given us in volume one our first complete presentation of Babylonian and Grecian mathematics. It is gratifying to find a scholar who lives up to the high standards he sets in our accompanying article. He does not lose his way in a maze of details, but portrays for us the evolution of ancient mathematical thought and exhibits the foundations on which our present knowledge of ancient mathematics is based. The volume closes with a discussion of Babylonian geometry and algebra; not until recent years has much been known about this subject, and Neugebauer has done more than anyone else to clear it up. Within a few years he has become one of the leading historians of mathematics and probably the most eminent living authority on ancient mathematics.

There remains the pleasant duty of mentioning Neugebauer's monumental edition "*Mathematical Cuneiform Texts*" which appeared in 1935, in two volumes. Space does not permit a full discussion of them here, suffice it to say they have been heaped with praise in all journals. Much of the material offered is interesting to the Assyriologist, the general historian, and the philologist. Architectural, engineering, economic and legal problems are touched on. The texts cover nearly two centuries and deal with the most diverse mathematical problems. One finds here a very complete bibliography of Babylonian mathematics.

G. W. D.

OTTO NEUGEBAUER

The History of Mathematics

By OTTO NEUGEBAUER
University of Copenhagen

Translation by Departmental Editor
G. WALDO DUNNINGTON

The historiography of a science usually does not enjoy any too high an esteem among its productive representatives. The reasons for this are several and they are not difficult to recognize, especially in a science like mathematics, which can distinguish with such precision between secured possession and unsolved problem. Mathematical problems and methods are indeed, like every other element of existence, historically conditioned, but that portion of the way already traversed—which for their continuation one must know and as such survey—is a relatively short one. Probably long centuries worked only in closest connection with antiquity, but the great rise of modern science begins with the appearance on the scene of entirely *new* ideas whose import lies in the opening up of hitherto unknown questions rather than in the settlement of old ones. In all natural sciences definitive results may have an absolute value—in mathematics a settled theory (the technical term for this is a "classical theory") is something dead, which cannot captivate fresh forces.

With this set-up the non-historical character of mathematics is not to be wondered at. In addition however, or rather connected with the above is the fact that the existing historical presentations of this science cannot interest the professional because their authors, as outsiders, with all their formal knowledge of subject matter, do not touch the real essence and interest of the problems. Instead one finds things treated such as the subject "Geometric forms that were in existence before the advent of life on the planet" (in a "History of Mathematics" appearing in 1923), which have nothing at all to do with mathematics. And only too often an anecdote collection, or even worse, an endless chain of priority questions must replace *history*.

It would not be worth while to write about such things, if one had to regard them as irrevocably united with the substance of the history of mathematics. Indeed I do not believe that the above mentioned purely objective relationship between progressing research and its history can be changed; but I regard it as an absolutely attainable goal, so to re-cast the history of mathematics *in itself*, that, like the history of philosophy, it will become an integral member in the series

of modern sciences and not lead to an entirely meaningless existence untouched by mathematical as well as historical spirit. Then one will again dare to hope that even the purely professional mathematician, from this broader viewpoint, can profit by occupation with it.

I have already touched on the point where according to my opinion the chief deficiency in the present condition lies: in the lack of an *historical-problem* attitude. I should like to explain somewhat more in detail by a very special question the way I desire this to be understood: What is the treasure of mathematical knowledge which the Greeks took over from the Orient?

Immediately an objection: What interest at all does such a question have? Indeed it is quite immaterial to know whether some Egyptian or this or that Greek possessed a formula for the volume of a truncated cone or not. And such an objection really exists quite properly, as long as one contemplates such knowledge only on account of its absolute content, but not as points of demarcation on a scale which one needs in order to be able to draw any sort of historical comparisons at all. However immaterial it may be in itself, whether these points of the scale are constituted by propositions of "elementary mathematics" or by propositions of an optional brand of modern analysis—that we have to deal with the one or the other is historically accidental—nevertheless the gaining of reliable factual material becomes important as a *preliminary* labor in order to have safe ground under our feet, and not to go to seed by merely attitudinizing esthetically.

Here I must interpolate a purely methodical remark which refers to the securing of the foundation for answering the question asked above. Our entire tradition from the ancient Orient exhibits one great advantage: we scarcely know *one* name of an artist or scholar. The entire cultural evolution of those periods is from the very beginning most closely united with the national unit, its picture is presented in much more tranquil lines before a larger background, than in an epoch when the struggle for "master or school" or for "genuine or false" distorts the perspectives. The chronological arrangement necessary for every historical comprehension must of itself ensue in a much broader framework: *in the framework of general history.* Thus with all the scantiness of our knowledge the history of Egyptian art, religion, and indeed of linguistic history forms with the "pragmatic" history a much more closely knit unit than is the case for analogous Grecian conditions. Directly therefrom arises however the demand to connect the utterances of mathematical thought with these general viewpoints. Not until the investigation of purely objective questions takes place

strictly on the basis of the history of civilization can one expect to attain a relatively correct evaluation of the separate problems. Then too such an investigation obtains, on the other hand, a much more general meaning because it is able to bring to light in a quite precisely comprehensible manner very charactertistic features of a people.

The disappearance of the individual in the history of Egyptian civilization has not always been regarded as an advantage. Thus the poor scribe "Ahmes" who immortalized himself as the copyist of the mathematical Rhind papyrus (the most important monument of Egyptian mathematics), has had to assume all possible titles from "king" or "teacher in an agricultural school" down to unskilled "pupil". Or however one has taken refuge in an intentional concealment of the individual behind the very popular "priest castes", although they do not exist, at least in the genesis period of all Egyptian sciences, thus it is maintained: "Mathematics as a science was in Egypt the exclusive possession of the priest caste and was carried on in the priesthood as an occult science and concealed from the people,"—with such success indeed, that not the slightest vestige of this "occult science" has been handed down to us!

To the demand for arrangement of historico-mathematical investigations in the scheme of general history of civilization is joined a sphere of questions much more difficult of access, as soon as it has to do with ancient history: the connection with philology. Such a fundamental investigation as *Sethe's*[1] book "On Numbers and Numeral Words among the ancient Egyptians" shows how much can still be summoned from these things for the beginnings of mathematical thought. One must not carelessly pass by these things, as soon as one asks questions about the historical development of the most important categories of thought, especially in a period of mathematical research like the present, in which the question about the logical foundations of mathematics assumes a central position. Here is really one of the points where the most uncognate sciences encroach quite directly on each other, so that it is not pertinent to grant philosophical speculation the only decision.

If one turns to the actual content of pre-Grecian mathematics, the first impression is that it has an exclusively "elementary" character, and is in itself rather homogeneous: simple problems of calculation,

[1] Ths distinguished Egyptologist, Prof. Kurt Sethe (1866-1934), liked to observe the phenomena of Egyptian culture and civilization from the viewpoint of general history and by comparison with other cultures, to assign them their place in the evolution of civilization. His work Über Zahlen und Zahlworte bei den alten Ägyptern und was für andere Völker und Sprachen daraus zu lernen ist (1916), as well as a number of his other papers, definitely enriched the history of mathematics.—G. W. D.

executed in part with the help of numerical tables, or problems involving the calculation of areas and volumes of geometrical figures such as those demanded by agriculture or at the most stonemasonry. But if one observes the role which these things played in their own cultures, this picture is quite essentially changed and offers problems which are by all means worthy of historical investigation. The deep-reaching distinction between the two great cultural units Egypt and Babylonia, even in simple numerical notation and application of the first calculation exercises, asserts itself here quite essentially. The beginnings are indeed in both districts the same: Hieroglyphics with special signs for the most important numerical values 1, 10, $^1/_2$, $^1/_3$ etc. and a purely additive counting foundation of all calculation. Now however the individual development sets in. The fundamental problem of Egyptian mathematics (which bears a purely "arithmetical" character) can be bluntly formulated: to provide those oldest *additive* methods with as sizable a domain of operations as possible. That which we today call "multiplication" is in Egypt repeated addition (by continued doubling and suitable collecting); indeed the entire fraction calculation, which in this form extended its influence over all of later antiquity far into our Middle Ages, owes its externally very intricate methodology only to the consistent execution of the same fundamental principle. We have here in abstract pure culture so to speak the same tenacious adherence to old traditional forms which characterizes all other portions of Egyptian life, which has made its theological systems a chaos so difficult to unravel. The inevitable transformation of religious concepts does not ensue by the formation of clear, new systems but by artificial interpretation of the oldest texts, whereby their simple meaning is distorted and (from *our* viewpoint) the most contradictory statements are entangled, rather than give up the fiction that everything has been thus from olden times.

Quite different in Mesopotamia with its much more stirring history. Even the further development of the hieroglyphic system created by the Sumerians ensues in an essentially different direction. While the hieroglyphic system in Egypt, at least as a system for inscriptions, was preserved to the latest period and while the hieratic writing represents only a levelling-off of it, nevertheless in Babylonia the hieroglyphic symbols, already of a marked linear style, were finally replaced by a number of pure "cuneiform symbols" which give up every conscious connection with the old hieroglyphics. Of course this is also tied up with external influences such as the inconvenience of clay as a writing material and the rise of new elements in the population. Consequently for numerical notation a system of figures very

deficient in symbols is formed, which approximate closely our present notation by "local value".[2] However the latter is again conditioned by a very early creation of independent multiplication (as is shown by multiplication tables and tables of squares preserved from a very ancient period) and a strong influence on the entire system of notation due to the standardizing of weights and measures. The details of this process are of course much too complicated for me to discuss here. As to general history it is stimulating to remark that the Babylonian system of *writing* ran into a cul de sac by preventing the gradual transition from hieroglyphics to "uniconsonantal symbols" and finally to "letters", that mathematics however from the beginning on pointed in a direction which in its consistent expansion by means of the Hindu local value system (with which our present notation by digits is identical) has furnished one of the most important supports for modern further development.

I hope one will be able to see even from these hasty references that the elementary, indeed frequently primitive character of the mathematical problems and methods of this early period, which has never been calmly admitted as such and efforts made to conceal it by phrases like "primeval Egyptian wisdom", results in quite the opposite by granting us an especially clear insight into the historical beginnings of mathematical thought. To be sure, one must waive the desire to build up a linear chain of mathematical knowledge from earliest antiquity to our time, but one must learn to place independent cultures like separate personalities beside each other. Then it will be recognized that every people and every epoch seeks in its own manner to meet the problems presented to it and the *comparison* of these various phases receives a new meaning. Not until Oriental mathematics in its singularities is really known, will one know how to evaluate properly those of Grecian mathematics and look for *those* points where specifically Grecian problem-attitudes set in.

But historical processes do not originate only by the encampment of various cultural types beside each other, for beside this "horizontal" articulating of cultures a "vertical" stratification plays an ever recurring role. The desire to regard such a complex structure as Grecian civilization as a unity would be a very fundamental mistake. Between the influences from outside enter the great differences between groups of the most varied intellectual tendencies. Pythagoreans, the Academy and the Sophists cannot be brought into one line of development, but stand in their mutual influence as independent simultaneous factors

[2] "Principle of position", or value due to position of the digits.—G. W. D.

beside each other. Thus one cannot co-ordinate the attitude of all these tendencies on mathemarical problems, even though the external result of occupation with mathematical questions is a permanent increase of objective knowledge. The really essential thing however is not the question whether one multiplies the square of the radius by 3.16 like the Egyptians or by 3.14 in order to determine the area of a circle, but to fathom the collective attitude on the problem of measuring the area of figures bounded by curves and the meaning of infinite processes to which this leads. And in the answering of *these* questions one will again have groups quite separated by principle to differentiate, groups whose transformations are to be pursued in detail, in order to gain a true-to-life picture of the entire process. And when the Arabs or West European civilization later tie on to the Grecian acquisitions, this occurs every time in a special manner and with preference for quite a definite method among these various currents, in spite of all continuity with reference to mere content. Thus the history of mathematics suddenly reaches out far beyond its narrower framework and offers no end of the most interesting questions, which reward the effort to review steps long archaic in mathematical thought.

And beside this general historical significance here comes to the front yet another which refers to mathematics in the narrower sense, but on that very account must not be forgotten. I mean, viz., that only *historical* thinking can possibly form a balance to the much deplored specialization. The last phase of our science inaugurated in grand style at the passage of the eighteenth into the nineteenth century, of whose universal character the great French scholars and the Humboldt brothers may serve as an example, has not been able to maintain this initial level. It is clear that a rigorous establishment of the newly unlocked sciences is to be accomplished only by the greatest division of labor in careful separate investigation. A consequence thereof was however not only a separation of the single sciences from each other, but also a crumbling of these disciplines themselves into divisions scarcely understandable or interesting to each other. There is no doubt that a serious reaction must be set in against this condition and in part already has set in in a very perceptible manner. The question about the whence and whither of a science, about its place in the broader sphere of our entire civilization, is being asked more and more decidedly. In all fields it is being shown that only in the *synthesis* of modern research methods with the less hampered perspectives of a deeper intellectual content can a guarantee for restoration of the unity

of all sciences be found. The work[3] by Felix Klein *"Lectures on the Development of Mathematics in the Nineteenth Century"* shows as does none other what the historical view in this sense can mean for mathematics. Truly historical thinking united with the most intimate research activity speaks to us here, reminding each one to understand and evaluate his own research tendency as an element of a great historical process.

It will not be vouchsafed to many to write the history of a science in this sense. However every single historical investigation can count as a usable *preliminary* performance toward further synthesis only if it is guided by two viewpoints: to see the history of mathematics in the framework of general history and to understand mathematics itself not as a collection of formulas to be continually increased, but as a living unity.

[3] Professors Neugebauer, Richard Courant, and Erich Bessel-Hagen (Bonn) prepared this volume for publication. It is entitled *Volesungen über die Entwicklung der Mathematik im 19. Jahrhundert* and appeared with the imprimatur of the publishing firm Julius Springer in Berlin, 1926, a year after Klein's death. These lectures cover a period of approximately 1914-1919 and were delivered by Klein to a small circle of students in his home. His death in 1925 brought to nought his plan to issue a more voluminous work on the subject, although this presentation fills nearly 400 pages. These lectures are especially charming because they are published just as he gave them, and never received literary "finishing touches". The book is a history of mathematical ideas rather than mathematicians, and an amazing example of his power of presentation; he portrays the train of thought leading to some discovery and as Professor Neugebauer has so well said above, he views isolated pieces of research within a larger framework. Klein's main thesis, running through the entire book, is that regardless of this or that genius a fixed and definite evolution of mathematical ideas exists. The science moves and must move ahead continually on this stream of development. The appearance of a genius merely hastens the current.—G. W. D.

The Teacher's Department

Edited by
JOSEPH SEIDLIN

[1]*The Place of Mathematics and Its Teaching in the Schools of This Country*

[1]II. COMMENTS BY TEACHERS OF COLLEGE MATHEMATICS

[2] *There is no transfer and it abides in mathematics. It is not without some misgiving that I publish, with and without interpretation, the many comments of our colleagues. For evidently teachers of mathematics, like unto any other group, hold to opinions and prejudices disregarding, and often despite, ascertainable facts. It is my hope that airing these opinions and prejudices in the open may reveal them to us as they are.*

I shall make no attempt to classify the comments or to distinguish, by special type or device, between opinions and prejudices and seasoned analysis and evaluation. The December issue of the National Mathematics Magazine will contain a summary and a general analysis of these comments.

Since anonymity has been requested by so many of the contributors, any possibly identifying statements will be altered in form but not in meaning. Needless to add, names, places, and other identification marks will be deleted or replaced by non-pertinent letters. Also, insomuch as many of our readers may want to refer to some of these comments, they will be numbered. It is my intention to choose the more significant and controversal comments and only occasionally insert one for "comic relief." I shall avoid duplications excepting instances of pretty or pungent remarks. Brackets [ ] *will be used consistently to indicate editorial additions.*

1. ... I am not at all sure that opinions collected at random by the questionnaire method ... have very much value. The judgment

[1] Cf. Teacher's Department, N. M. M., No. 8, Vol. 10, May, 1936.
[2] With apologies to our educationists and Nietsche.

of one really informed person[3] (e. g. A, B, C, or D) is worth more than that of a thousand "zeros". As I know teachers, I feel that very few of them have ever given more than passing thought to the basic reasons for teaching mathematics. . . .

2. . . . To my mind, it is not so much the "superior" or "scientifically minded" student who should be urged to study mathematics beyond geometry (he will naturally decide to do so of his own volition) as it is important that students who never hope to go on with scientific studies should be advised early not to discontinue the study of mathematics.

At "H" mathematics is not required of all freshmen. But we advise those hoping to study physics, physical chemistry, or statistics that at least our freshman mathematics course is practically an essential for their future work.

Since a few high school students entering upon their eleventh year know whether or not they are going to college, or what type of work they will be pursuing when they go to college, it should seem worth advising them to continue the study of mathematics unless they are sure they are not going to have occasion to use it.

I should say that the average student, with some effort perhaps in certain cases, can take with profit the courses usually offered in the eleventh, twelfth, and thirteenth years. These are not intended primarily for the "superior" or "scientifically minded" only.

On the other hand, I have met very intelligent students, for whom courses in mathematics beyond geometry were a real task and largely a waste of time. They were logically minded and could reason about things quite as understandingly as scientifically minded persons. But the subject matter and the symbolism of mathematics simply did not appeal to them.

3. . . . All of our trouble has been caused by false prophets in our colleges of education. Their philosophy is one of gaining popularity by preaching doctrines which advocate the elimination of work from both the high school and college curriculum. . . .

4. . . . Would call a high school, college, or university education in the liberal arts division without a proper amount of mathematical training as defective. The assaults on mathematical teaching as a general cultural subject are sponsored by educational molly-coddles. . . .

5. . . . I am glad to see that others are beginning to complain about the way in which mathematics is being taught in the secondary

[3] It so happens that the "judgments" of A. B. C. and D. are included in the following comments.

schools today, particularly in the publicly supported schools. Mathematics, for instance, in "V" schools is almost entirely optional, and as a result many of the students take the lines of least resistance.... The trouble, however, does not lie entirely with the students. The failure on the part of principals and other high school officials...accounts in a large measure for the lack of preparation which is apparent on all sides....

6. ... I do not place much stress on mathematics which is *required of all.* The presence of students who are working for a grade only lowers the standard of all.

It is strictly up to mathematicians as a whole to make mathematics really and obviously worth while to all students. I would make drastic changes in content. Our belief that it should be required will exercise very slight influence on Boards of Education and Superintendents or faculties of universities. Anything all mathematicians put together can say will have little influence. Our judgment is not supposed to be unbiased. When we as a whole make our work really worth while to all students we will not need to argue the question. But that is still a long story....

7. ... I am happy to answer this questionnaire. The trouble is not the subject so much as the teacher. We have high school teachers of mathematics in "P" who think it a waste of time to study mathematics....

8. ...eventually there will be different types of mathematics courses in high schools and possibly in colleges....

9. I am of the opinion that too much mathematics is taught as though it were a necessary evil, such as taking spring tonics.

Many of the criticisms of mathematics in the high school and college are not so much on the subject as the subject matter and the routine uninteresting way in which it is presented.

The school of education here or rather some of its faculty occasionally direct a blast against mathematics. I think, at the present time, the majority discount this as propaganda, but it is like the continued advertising of, say, "Pink Pills for Pale People". Eventually it is going to wear down the general public's resistance unless we do something about it. I am glad that you are starting something. More power to you and count on me to do what I can....

10. ... The common notion seems to be that all education should be utilitarian. We send youth to school and to college that they may the better succeed in the game of competition. Is that all there is to education? I answer, NO.

As a rule the home and the pupil do not know what is best for them. The school teacher stands in the position of doctor, who knows what is good for health. The school should prescribe and not leave taking of medicine to choice. I believe that there is a distinct correlation between our present day educational program and the capacity of our public servants, as expressed in their performance in public office. The scientist is the most dependable man we have today....

11. ... I believe that in many places the so-called improved content has not been the desired improvement... I believe more attention should be given to formulating mathematical courses which will give the foundation for the mathematics that teachers of elementary grades should impart to their students. ... If they (the prospective teachers) were required to take a college course in arithmetic as well as the course in "How to teach in the elementary grades", there would be a larger and better crop of mathematical enthusiasts entering high school....

12. ... Ten years teaching experience has led me to believe that mathematics properly taught is both enjoyable and profitable for any student who is not definitely deficient mentally. The greatest difficulty in teaching college mathematics is due to improper early training. If the training of students in the early stages of mathematics were restricted entirely to the fundamental operations of algebra with the total exclusion of such formal "rules" as "transpose and change sign", etc., not only would the teaching at the college level be easier but also the interest and enjoyment of the student would be greatly increased....

13. ... I see little value in teaching the *technique* of algebra to those who will not use it as a tool. However, experimental geometry, mathematical method of thought, the human significance of mathematics seem to me to be worthy of a place in the general curriculum....

14. ... I believe that it will be unfortunate to require *all* high school pupils to take a year of algebra and a year of geometry.... To require these of all is to force into the classes of those who can secure real improvement a considerable body of incapables and thus to make it less valuable for the former group....

15. ... I am strongly in favor of the old reliable plan of teaching hard mathematics in high school and college. The idea of making everything easy does not appeal to me or to the average student....

16. ... The difficulty arises because in many states the school appointments seem to be made without any regard for the preparation of the teacher. There is no lack of well prepared teachers of mathe-

matics, but all too often a teacher of English or gymnastics is given the responsibility for mathematical teaching. It is a vicious circle: badly trained boys, taught by teachers with neither knowledge nor love for mathematics, grow up and enter legislatures where, most logically, they vote against requiring mathematics in the schools....

17. ... In the high schools of "K" bad teaching and bad learning are a consequence in large part of the *ignorance* of the teachers. The improvement of content will have to wait upon better prepared teachers. Professional pedagogical theory tends to identify secondary school mathematics with manipulation.... The teachers do not seem to have the required knowledge to present secondary school mathematics as a science rather than as an exercise in mnemonics....

18. ... My answers [to the questionnaire] are subject to my belief that all elementary mathematical content should be redistributed so that much of what now parades as *higher* mathematics would be introduced, in modified form, much earlier; and some of our present ninth and tenth grade mathematics would be postponed....

19. ... No one should be *required* to take mathematics, but a system of education which does not convince the pupil that he must have a knowledge of mathematics in order to interpret the civilization of which he is a part has failed....

20. ... The trend in education now is to keep away from anything that seems difficult to the student. Latin has gone, modern languages are on the way, and science and mathematics are going too. Soon there will be nothing left except Agriculture, Shop Work, Domestic Science, Social Science, and last but not least Education.

This college requires twelve semester hours [of mathematics] for a degree. The students can and do get it, and after it is over are glad they had it....

21. ... It was true then, and it is also true now that the average grade teacher and high school teacher knows little or nothing about mathematics. ... I am not exaggerating when I say that we have students in the engineering school, bright enough and capable, who have never learned the multiplication table or for that matter any of the fundamental operations. The trouble with school officials is that they are all too ready to try all sorts of fads but make no effort to improve the quality of the teaching....

22. ... American education follows the cigarette slogan "It is mild and it satisfies". Mathematics cannot be successfully taught unless there is a spirit of serious study and clear thinking—something lacking in our high schools and liberal arts colleges....

23. ... There are some [students] who are unwilling or unable to think logically. Such underprivileged students will never be educated. They do not have the most important characteristic distinguishing them from the other members of the animal kingdom....

24. ... I would hate to have to teach mathematics to *all* students of a liberal arts college ... Strongest conviction: Genuine, solid courses in mathematics as provided by our better text-books should be available—without dilution or sugar coating—for the good students. I do not care what they do with the morons....

25. ... However, I am also of the opinion that what we frequently term the lack of mathematical ability is the result of poor teaching and faulty instruction in the grades and in the high school....

26. ... There are few if any subjects which require the precision or the analysis which mathematics requires. To be able to work with precision and to analyse a situation are abilities which everyone should have the opportunity to cultivate. The non-scientifically-minded are the very ones who most need the kind of mental training mathematics offers....

27. ... All college courses should be elective, except as required for the pursuance of a professional career. It is sheer waste of time to attempt to teach a college student against his inclination for purely "cultural" purposes....

28. ... If we could stop the present pernicious custom or practice of permitting a student to continue with third grade mathematics before he has mastered second grade mathematics, then a great part of the fancied difficulty of lack of special mathematical ability would disappear....

29. ... In my opinion, the burden lies with the teacher. I cannot believe that mathematics need be uninviting to students of any ability in view of the universal interest in puzzles, games, etc., that are essentially mathematical. We are to blame for the decreasing interest in that we have failed to exploit the best of our subject. I believe that of all subjects ours, as taught, is most pedantic. This, I believe, is due to the fact that, contrary to general opinion, mathematics is a refuge for an inferior type of teacher....

30. ... In college (freshman year) a five hour introductory course should be offered for those who do not want to go on with mathematics. They should take it for what they can get out of it. It should not be a course in which a student can "flunk", i. e. all should pass....

31. ... I do not believe that anybody who has had actual experience with the type of people now found in the ninth and tenth grades would think for a moment of "requiring" that *all* these pupils take the standard courses in algebra and geometry generally offered in those years. Were these required of all, and proper passing standards maintained, the per cent of failures resulting would not be tolerated by school administrators. When the dull and the slow-normal are forced to take the ordinary courses in algebra and geometry, the courses must be so diluted and the teaching so slowed down that the bright pupil—perfectly capable of doing worth while work in mathematics—is discouraged and disgusted... Finally, I believe that every ninth and tenth grade pupil should take mathematics courses fitted to his needs, abilities, and interests; the bright pupil, courses in real mathematics, algebra and geometry; the dull and the dull-normal pupil, courses that he can do with success and that patch up some of the holes in his intellectual equipment in arithmetic.

32. ... I would change question 3 [see questionnaire in May issue] to read: "Do you believe that mathematics at the level indicated below should be such as to require the teacher to give special attention to the improvement of the student's 'mathematical ability'?". Such a requirement would perhaps necessitate the elimination from the mathematics class-room of teachers untrained in mathematics. I believe that "mathematical ability" is a composite of abilities of the trained mind, and that, instead of considering these abilities as "special", we should seek to develop them in all people....

33. ... If a high school student does not have the mental capacity to pass a course in algebra and in plane geometry,I do not see that he has the mental capacity to profit by a four year sojourn in high school. Such a student will emerge just as ***uneducated*** as when he entered, unable to think clearly in any direction. Even if he cannot pass such a course (algebra and geometry), he will gain more by trying to pass it than by avoiding it... I doubt if any but the subnormal would fail to develop some mathematical ability if they had been properly handled from the first grade upward....

34. ... If the first college courses in mathematics are college algebra and trigonometry, I would require only the mathematics and physical science majors to take them. If the first college course in mathematics is a unified or general course, I would require it of the majors in biological sciences and economics. If the first course is one including statistics and mathematics of finance, I would require it of practically every student....

35. ... I believe that algebra and geometry at the high school level should be ***rigorous*** and should adhere closely to the classical content of the course. I am opposed to have these courses oversimplified and to the omission of important topics....

36. ... I would reword question 1(a) to read "Do you believe that at least one year of algebra and one year of geometry, as taught to you thirty-five years ago, should be required of all high school pupils?" My answer would be an unqualified "Yes". It seems incredible to me that the teaching of mathematics in earlier grades has declined to the point portrayed by some of our members at the recent meetings.

As you may imagine from my answer to the first question, I appear to be a reactionary. How can I be otherwise? All the algebra and plane geometry I studied before entering college, I had in the sixth, seventh, and eighth grades before entering high school. I was an ordinary lazy student like the rest of the boys in that school. We had a man teacher who made us all eager for algebra and geometry, especially algebra. The only mathematics I took in high school was trigonometry and solid geometry. This will explain why I hark back to the "good old days". It was of course a restricted curriculum but there was fundamental discipline and training which modern educators ignore today. ....

37. ... It would be impossible to cover adequately the points which you raise without writing a letter of a dozen pages...

If the high school population and teachers were of the same calibre as thirty years ago, I should vote "Yes" on 1(a). As it is, I do not believe that half the pupils in high school are fit to be there and my impression of the teaching profession, derived from the published utterances of their spokesmen, is that they are a bad lot. Therefore my first reaction to your question was to say that, in my opinion, the study of mathematics in high schools should be ***absolutely prohibited***. This drastic action is probably not feasible. As a compromise, it might be possible to do this: Have a qualifying examination in arithmetic at the end of grammar school, permit only the first thirty or forty per cent to take mathematics, and restore algebra and geometry to their old standards,—in other words cut out the boon-doggling.

My answer to question II is "Yes", if any subject is required; a negative answer is tolerable only in the event that no subject is required. Personally, I believe that the best situation is what we had here once when every freshman had to take either Latin or Mathematics

and, if he took Latin, had to take Mathematics, Physics, or chemistry. For the present, I think that the best solution is to limit rigidly the number of freshman courses and to stack the cards so that for about half the class mathematics would have to be chosen to satisfy group requirements. All this applies to men's colleges; I have no interest in women's colleges or in women students of mathematics, except a perhaps a negative one.

Nearly all the trouble which freshmen that I have met have had with mathematics is chargeable to past and present unwillingness to work and to improper earlier training. You have probably seen that book "A" and myself, which is a moron's delight if there ever was one. We use that with the freshmen and so I am teaching calculus to men who have no algebraic sense whatever; it is really funny when you overlook the unpleasant side. I do not believe that 10% of this class could pass the college entrance examination in algebra which I took for admission. Yet this class represents about 40% of the whole freshman class who have elected mathematics, presumably because they believe themselves better qualified in it than in other subjects....

... My personal belief is that efforts to save mathematics are doomed to fail, until the present retrogression of the American people towards barbarism is stayed. This has been going on since 1900 and I see no signs of a reversal. The attack on mathematics is merely a small part of the war against excellence of any kind....

38. ... In the past we have used mathematics as a *sieve* with which to sift college "timber" from non-competents. Although the content and method of teaching mathematics need improvement, the discontinuance of this sifting use for it will do much more for its rehabilitation than anything else. With improvement in the tests to determine ability we no longer need so imperfect a tool for this purpose as mathematics has proved to be.

39. ... I feel that the tendency for secondary mathematics to lose ground is due to the conservatism of the body of secondary school teachers. Their failure to keep the subject matter modernized has caused it to seem irrelevant to popular experience. Their chief defense is based on transfer of training. Transfer of training may exist, but its defense is not convincing.

If the formal synthetic methods of plane geometry were replaced largely by analytical methods, high school geometry might cease to be such a mental barrier.

If the preponderance of plane geometry problems and of grocery store puzzle problems were supplemented by pertinent problems from

the fields of social and physical sciences, freshman mathematics in college would not need to be required. It would be demanded.

40. ... I do not believe that persons who are made very unhappy by the study of mathematics should ever be forced to take it. But young people should be made to understand clearly that many lines of activity will be closed to them if they do not learn the beginnings of mathematics. My dissatisfaction with the present status of mathematics in our curriculum is largely due to the fact that non-mathematical advisers of students do not themselves clearly understand the situation....

41. ... I would not expect every college student to take the usual courses in college algebra, trigonometry, and analytic geometry, but only such portions as are needed for a fair understanding of graphs, of the nature of stocks, bonds, annuities, insurance, compound interest, etc., so that the student would have at least some idea of what he was doing when he bought insurance, or an automobile through a financing concern, or a home through a monthly payment plan....

42. ... Mathematics is meat for strong men, not milk for babes and sucklings. Let us frankly admit it and not seek the unfit.

We must admit the trend toward quantitative thinking in the social sciences... Educationists emphasize the social sciences in our curricula and yet, paradoxically, refuse to give the students the tools for understanding them. They cry "Social Efficiency" as a shibboleth and yet build a curriculum that leads to social inefficiency....

43. ... While I do not believe that the successful study of mathematics requires either superior minds or especially scientifically-minded students or especially mathematically-minded students yet I do believe that in all grades we are likely to find some students, not at all a large percentage but large enough to be significant, who can study mathematics successfully only at the cost of unusual effort. I do not believe that these students will profit enough from the study of mathematics to make it worth their while.

If two years of mathematics is to be required of all high school pupils, I am inclined to favor differential courses... Of course, this opens up the whole question of differentiation in education.

So far as the traditional content is concerned, I am of the opinion that it may be possible to develop an equivalent course of more significance than the present courses in algebra and plane geometry.

44. ... I do not feel that forcing students who have already acquired a distaste for mathematics to take more of it is the best approach to the solution of our problem. I think it is safe to say that

human beings come into this world with no more prejudice against mathematics than against any other study—certainly any other study that requires thinking. But sometime between the first grade and the twelfth grade a very large percentage of those who enter college have developed a distinct dislike, for mathematics. If you can get at the cause of this situation, better results might be hoped for than by making mathematics in college compulsory.... Just as there is a high percentage of entering freshmen with a dislike for mathematics, there is also a high percentage of seniors and graduate students who lament the fact that they did not take some, or more, mathematics.

45. ... No one should be allowed to teach mathematics except a real mathematician who has had at least a year of graduate work in mathematics... Even students not preparing for college should be taught by teachers who have the qualifications mentioned above and in addition have a much broader outlook on life. This group of teachers should be thoroughly versed in the history of mathematics and thoroughly familiar with history, economics, physics, and chemistry. In addition they should be familiar with business practice....

The content of courses in mathematics should be selected by mathematicians who are interested in education. In no case should a person who is an educator, interested in mathematics, be allowed to to have a word to say. It is the latter group who have ruined our courses in mathematics and are still engaged in ruining them. They know just enough mathematics to fool people into believing they know more than they do, and not enough to be of any use. A person who cannot acquire a master's degree in mathematics in our best institutions has no business to teach mathematics in any institution....

46. ... While I should not oppose changes in algebra and geometry aiding their presentation, I would oppose changes which went to the extent of replacing traditional courses in mathematics by superficial vocational or trade courses in the narrow application of very limited portions of these subjects....

47. ... I think there are people to whom both a high school diploma and a college degree should be available without mathematics. I have in mind the type of person who has real ability in other lines but who has a blind spot in the direction of mathematics.... Unfortunately concessions to such open the door to a relaxing of standards in general. Emphasis must be maintained on the "real ability in other lines."

On the other hand I believe it is of the greatest importance that we provide ample early mathematical training for those who have a talent

and an ability for it. It is important both for supporting science in this scientific age and for providing an intellectual culture of the kind best adapted to their needs to those who enjoy mathematics.

Between these two extremes there is a sort of average group, perhaps the largest, to whom mathematics is not impossible, at least at the high school level, but who show no aptitude in it. For them it seems to me that mathematics, perhaps through plane geometry, is certainly as worth while as the proposed substitutes for it; that it does offer a certain training in methods and study habits (yes, I know of the investigations in its non-transfer values), and an acquaintance with some of the methods of thought of really educated people. I do not see that the average pupil can spend his (or her) time better.

It appears desirable that these different groups, and perhaps intermediate gradations, should be separated for different treatment. When should it be done, and how? I do not believe there is any satisfactory method except to have the pupils actually try mathematics. I do not believe that mathematics shows much as to the ability or liking in later mathematics. In my own case I disliked arithmetic, liked algebra reasonably well, but did not feel any particular aptitude for mathematics till I reached geometry, and I do not think this is unusual....

48. ... But if he is going to specialize in languages or history, I think he should be allowed to omit mathematics. True, he will miss some valuable training, but if he wants to miss it that is his privilege. Before he is allowed to omit it [mathematics] he should be thoroughly informed as to what it means, but after that the choice should lie with him. There will be mistakes, but I know of no safe way to keep people from making mistakes....

49. ... If courses which emphasize insight rather than technique are offered, I think every student in the high school should be exposed to mathematics for at least two years. Why should we send our young people from the schools illiterates insofar as functional quantitative thinking is concerned? ....

50. ... In my experience "special mathematical ability" is rare. I have never met but one person who seemed to have such an ability. I do believe that education at each of the levels indicated requires progressively increasing ability to think. This ability can be cultivated, but not instilled. This does not mean that I am condemning the American ideal of education for everybody, but only am advocating that the type of education be adjusted to the student.

Beyond the introductory college course the liberal arts student should be allowed to follow his own chief interests. I would substitute for the adjectives "superior" or "scientifically-minded" the adjective "genuinely interested". Wherever I discover "mathematical ability" in a student, I find by consulting his instructors that this ability is exhibiting itself in all of his other subjects in which he is interested. Interest depends to a large extent on the instructor....

51. ... If we could count on the teachers of mathematics endeavoring to develop an interest in it, rather than treating it as a subject of discipline, I think that we would obtain much better results....

52. ... Mathematics as now taught in the high schools of this section is of no great value. The tendency to take all of the hard work out of the high school curriculum and to pass all members of the class has been carried too far, at least in this part of the country....

53. ... My impression is that the tendency is to make mathematics too formal. For many students it is allowed to remain a mere "juggling with figures". To correct this meanings must be insisted on.

What everybody needs nowadays is careful marshalling of facts and drawing of valid conclusions. This is mathematics regardless of subject matter. Almost everybody can think logically if they have clear concepts of what they are thinking about, but it is reprehensive to allow them [students] to grow up without learning to be critical of facts and of the processes of drawing conclusions....

54. ... Every student capable of proceeding to a next higher grade should receive training in the right way to investigate problems to weigh results, to decide upon wise use of methods. Mathematics is not the only subject in which this can be done, but it is the subject in which the materials used are simplest, and the mathos most obvious; hence it is desirable for each grade.

55. ... At present and as taught I believe geometry is largely a fruitless "memory" subject and algebra a mysterious thing without any connection with reality....

56. ... As a department, with five members, we agree unanimously with the sentiment expressed at the St. Louis meeting, namely, that mathematics is in a very precarious position, and is losing ground. When men like "A" go around the country spreading propaganda such as "When you move from B to C, you forget all the telephone numbers you knew while you lived in B, and that is just the way it is with your mathematics", and froth of that type, much harm results. In our Arts College, with an enrollment of 184 students, we had only one student taking calculus last semester. Our Engineering and Science

students respect mathematics more and more; our Arts College students less and less....

57. ... I think many acquire a distaste for algebra and elementary geometry because they are required to begin them too young. The same students a year or two later will learn easily and enjoy the work. Frequent aptitude tests, without stigma for failure, would solve this problem....

58. ... However, I do believe that mathematics is a subject in which special aptitudes exist, positively and negatively, in a more definite way than in most subjects. I see little advantage in forcing high schools pupils with zero or negative aptitude for mathematics to grind through algebra and geometry....

59. ... It is my opinion that any normal student can easily master all the mathematics required for the A. B. degree in a Liberal Arts College, i. e., at least six semester hours, and that any reduction of this requirement violates the fundamental principles underlying the definition of the Liberal Arts degree....

60. ... In other words, the avoidance of mathematics should carry as a penalty the necessity for taking a subject of comparable difficulty in the scientific field....

61. ... I should like to see every pupil make an earnest effort to learn "first year algebra", the facts of geometry (so-called informal geometry, and acquire an appreciation of the logical implications of demonstrative geometry. ... I would be ready to excuse any unhappy pupil after he has made sufficient earnest trial (I cannot define this, but I think teachers can judge pretty well) of each of these three. On the other hand I think it unwise to make it easy for bright pupils or average pupils to avoid algebra or geometry simply because they are lazy or have a mild antipathy to the content of these courses....

62. ... Many years of experience in teaching mathematics in college and in administering admissions have convinced me that it is a waste of time to force a student who has an active dislike for mathematics to continue in that subject.... In theory I believe in a revised or improved course in algebra and geometry for the first year; in practice the average school can probably do no better than to offer elementary algebra in the ninth grade and plane geometry in the tenth....

63. ... There are some to whom mathematics at any level will always remain a sealed book. But I believe the student of average ability, if interested in mathematics, can go through the calculus with profit....

64. ... I recognize that most of the elementary teaching has been very inefficient because the teachers did not have sufficient preparation nor perspective... Text books have been denatured and expurgated of very vital material.... It is disheartening to think of the time spent in arithmetic before the high school period, and see the result of it....

65. ... All students need training in careful thinking as opposed to guesses, hunches, inspirational notions, etc. Horse sense and a fair I. Q. and a lack of mathematico-phobia are the only requirements....

66. ... I have had the chair of mathematics in this school for more than a third of a century. I have met and taught hundreds of students. I have observed their lives after they left college. I find that nearly all of those who tackled their mathematics in the right spirit and learned to reason from cause to effect, who really learned to think, have given a good account of themselves in later life....

67. ... Much of secondary mathematics, as now taught, is mechanical and relatively meaningless. Such mathematics is certain to disappear from school curricula if other subjects prove more worth while...

68. ... I believe that any good teacher of mathematics, given the power to select mathematical material appropriate to the group under consideration, could organize a course of greater interest and value to the group than many courses which are now required of them in other departments....

69. ... The professional educators are partly responsible for the present situation in mathematics. They claim that they have performed "accurate", "scientific" experiments which show that training in mathematics is of no greater value mentally than training in sewing, etc. In reality, while the experiments have been performed for the purpose of getting exact numbers in mimicry of the more exact sciences, the conclusions drawn are exactly opposite to what these "numbers" indicate. But the spreading of the "results" of these experiments by professional educators all over the land has helped to put foreign languages out of the currciulum and is helping to influence the mathematics requirements.... Last year the mathematics requirements of seven semester hours for an A. B. degree for elementary teachers in the state of "K" was removed and the requirements in Education were raised from 18 to 30 hours....

70. ... Personally, I have used large amounts of time with patience and deeply thought concern to learn for myself whether so-called dumb students in mathematics could learn mathematics. Not

once have I failed to get uniformly good results. I feel therefore that any college student can learn the elementary mathematics if properly taught....

71. ... In any event it seems quite clear to me that any insistence on uniform requirements of mathematics for high school and college study is futile, and that a much healthier situation will arise if we allow these subjects to find their constituency in a very different way. We shall then get better students. There would be less complaint about the character of the work, and many who are now wrought up by the talk that is going on and who are kept from doing cheerful and effective work by the very poison that is in the atmosphere, might become cheerful and effective students and teachers of mathematics much to their own benefit....

72. ... The worst tendency that I have noticed in the mathematical instruction in the secondary schools is that the majority of the text books seem to be written by teachers of "education". I think the schools of education are having a detrimental influence on the instruction in secondary mathematics. I suspect that the sad plight of mathematics in the secondary schools (and colleges) of the middle west is due to such influence....

73. ... I am sufficiently convinced of the intrinsic worth of mathematical training even for the student who does not expect to go to college. But we must get rid of a lot of junk before we can urge this with a straight face....

74. ... The evidence is unanswerable that geometry as usually taught gives the average student neither the power to handle concrete geometric situations nor a transferable ability to draw valid conclusions from hypotheses. But there is real value in geometry... One of the most vital problems we have to settle is how to revise our teaching so as to get the results we know we should have....

75. ... One thing that ought to be emphasized more than it is, is that, unlike most other subjects, mathematics cannot be "picked up" outside. The student who fails to get it in regular courses usually never gets it at all....

76. ... When the foes of mathematics use the clear cut reasoning from certain well-defined and accepted hypotheses to carry their point, then in reality they are not foes but friends. But most of them have been reared on thin intellectual soup and their reasons are as thin as their own intellectual capacity.

77. ... I am convinced that plane geometry, using, say, "W's" text book, is a waste of time....

78. ... Twenty-five years ago I did believe that any student, with right text and teacher, could do the mathematics through the sophomore year at college. But my ideas have been modified by experience.... A professor of Biology and author of many text books in his line was refused graduation because he could not pass work in trigonometry. He came into my class and ***I passed him.*** I believe that curriculum requirements and the rules at the dean's office should be flexible.... No doubt the subject matter of mathematics can be be and should be improved. In properly modified form, and allowing for exceptional cases, it should be required at least through the tenth grade....

79. ... The so-called unified courses in mathematics are unified so far as the teacher is concerned, but the students usually fail to see the unification. As far as we are concerned they have not been successful. I suppose courses in mathematics should all be revised so as to make them practical, real, and vital at the same time. As yet I do not know of any such courses....

80. ... If I were running a high school I should give all students preparing for college algebra, more algebra, and still more algebra. The weaker the student and the more he lacks "special mathematical ability" the more algebra he should take.... The average high school student is not qualified to understand demonstrative geometry. In fact, the superior college senior is not so qualified; neither is the average high school teacher of geometry. Only a few high schools teach trigonometry so that it does not have to be taught all over again....

81. ... I lay a good deal of blame for the growing unpopularity of mathematics in high school to the seeming absence of social significance in the content, especially, of high school geometry. Notice that I say ***seeming.*** Teachers dwell upon the letter of geometry and forget the spirit of it....

82. ... Competition with other subjects must be met. This can be done but not in the old way.... Even other [other than superior and scientifically-minded] folks can be made to see some of the beauty of mathematics, some of its problems, some of its thrilling applications, but not many can be made to do so much of the technique as now taught and which in some cases constitutes 75% of the work....

83. ... I raise the question whether some of the courses in elementary algebra as now taught can be called mathematics....

84. ... I have no way of proving that "special mathematical ability" is the myth I think it to be. I think "s. m. a." is a misnomer for "proper previous education". If a student has not had "p. p. e."

he should be shunted away from mathematics in college or, better still, shunted away from college. . . .

85. . . . Only those who honestly pass high school mathematics should be permitted to enter a college of liberal arts. The others should be urged to take up a trade. . . .

86. . . . Any highly intelligent person has mathematical ability. It occurs to me that the arguments set forth by those seeking to discredit mathematics as a school subject could be used for the same purpose with regard to almost every subject taught in high schools and liberal arts colleges. . . .

87. . . . On the whole I think "dilution" and so-called improvement have gotten to be a fad. As a class, we better let people's mistaken notions take their course, put ourselves in the crucible, and in twenty years come out purged. Wishy-washy mathematics is worse than none. . . .

88. . . . I wonder sometimes if the modern trend has not gone to an extreme in assuming that pupils in the grades, in high school, and in their first year in college, are prepared to know what they should study. Perhaps our courses should be so ordered as to help the pupil to such a decision. . . . Do survey courses such as we have here at "B" help to make students more appreciative of mathematics?. . . .

89. . . . I do not think that mathematics is losing ground at the college of "C". The department is more than holding its own. . . .

90. . . . I think a sound course in Plane geometry is the best all-round course in our modern educational curriculum just as it was in Euclid's time and in all the two thousand years between. . . .

91. . . . I doubt the worth of geometry; unless treated in the analytical method. . . .

92. . . . For general use in high school and also in college a course about mathematics must be developed to inform the future citizen of the place of mathematics in his world without expecting him to master or even to be interested in its technicalities. . . .

93. . . . I fear I am hopelessly in the minority. Since Latin and Greek are gone, must mathematics go too!

94. . . . I have had a few students with marked ability in history, literature, and foreign lagnguages to whom mathematical thinking remained foreign. Still I believe that even such students should come in contact with mathematics. . . .

95. . . . I believe that every bona fide student in a liberal arts college should have some knowledge of the calculus. . . .

96. ... Too many high school teachers in mathematics have been trained in teacher's so-called colleges under teachers who know the latest educationist vocabulary but very little mathematics....

97. ... I believe the greatest menace of the present time is the loose thinking of those who are supposed to be educated. The A. B. degree, available to everyone, regardless of any training in accurate thinking, has become quite meaningless....

98. ... The whole trouble lies in teaching everyone exactly alike. We should test and group so as to give the better ones more mathematics and the poorer ones more drill and projects in the use of easy methods. Obviously, to omit is not to solve the problem of mathematics in curricula....

99. ... I believe the theory that students should do only those things they like to do has a great deal to do with the present unrest. We should try to get them to like to do those things that they should do....

100. ... Our college once had a mathematical requirement for a degree. Fifteen years ago it was abolished. Since that time, with no increase in the general enrollment, my classes in calculus have have grown in numbers and ability.... Mathematics, properly taught, can take care of itself without being damned by being put on the required list....

101. ... In my opinion quite all normal minds are capable, under proper guidance and inspiration, of learning to effective advantage to themselves an amount of mathematics equal to twenty-four or more semester hours of a standard college curriculum....

102. ... If and when a six hour course about (not on) mathematics, that gives a conception of the nature, history, and place of the subject, can be worked out, it should be required in a liberal education....

103. ... The needed "special mathematical ability" varies inversely with the mathematical background, training, and teaching ability of the instructor....

104. ... School boards should be required to appoint teachers of mathematics on the basis of knowledge of the subject and not on the basis of credits in methods of teaching....

105. ... I believe that we [college teachers] need not be especially concerned with problems of the high school. Let them [high school teachers] take care of their messy situation. We have our own.

106. ... When I entered college in '92, the entrance requirements were definite. Since that time they have been "broadened and enriched" but in my humble judgment not improved....

107. ... Probably two-thirds of all "well educated" people will find use for at least high school mathematics in their life work. Perhaps 10% can use four years of college mathematics. Almost anybody, with adequate training, can get from mathematics the satisfaction comparable with that from music or poetry. But some people have no "ear" for music, and some never learn to enjoy mathematics. Those who do not know whether they will undertake a life work needing mathematics, should be told that it is better to learn it young and not use it than to need it later and be unable to get it....

108. ... I should like to see 3 hours or 6 hours of mathematics required, but totally different from any now taught in any college I know....

109. ... I do not believe that college students should be compelled to elect mathematics. But the usual freshman course should be so revamped that most students would want to take it. Our present courses are largely designed for those who expect to continue with mathematics. Most people prefer to live in a completed cottage rather than in the first floor of a roofless first story of a building which was supposed to go up four stories. The analogy is obvious....

110. ... We have some students entering this college without either algebra or geometry. Their choice of courses is limited and they are greatly handicapped. You should hear them talk about the high school that permitted them to graduate without mathematics....

111. ... Students of college grade, who like mathematics, should be granted the privilege of studying it, not required to do so....

112. ... Under a fairly free elective system, I believe mathematics should (and can) manage without special protection....

113. ... Sometimes a genius will appear who will frame the course that ought to be taught to all students in colleges....

114. ... Some pupils in high school will profit much more by arithmetic, handled with some algebraic symbolism, than by the traditional algebra and geometry....

115. ... Any school or college subject, where accuracy and thought, not memory, are required, is fully as difficult as mathematics...

116. ... I am hoping that we may improve the teaching of mathematics for cultural ends and give the greater part of high school students the valuable educational training that is possible from such a course....

117. ... Students entering the State University are so strikingly deficient in thinking power that the question of high school curriculum seems relatively unimportant... but much of present day teaching of mathematics in the high school is of slight if not of negative value (negative, i. e. if one has certain ideals)....

118. ... Poor teaching gives high school mathematics its "black eye". When well taught it excels all other subjects as a stimulus to intellectual curiosity....

119. ... The study of mathematics, like that of any other scientific work, presupposes the ability to reason. It requires *mind* but not a special mathematical mind....

120. ... Little "discipline" remains in education. Mathematics affords the best. Let us keep it as imperatively needed....

121. ... We used to require six hours [of mathematics], then reduced it to three and now none. I wish that more students would elect it, but I do not know how to bring it about.... After forty-five years of teaching I know much less than I used to about such things....

122. ... I have taught in both systems [required mathematics and elective] and I have found the elective system more advantageous. The presence of poor students causes the calculus and subsequent courses to deteriorate beyond recognition....

123. ... I believe that "special mathematical ability" is required at any level, but I have in mind special mathematical ability of the teacher rather than the student....

124. ... Hope you get enough ammunition for a good shooting up of the Pedageese....

125. ... All I am certain of is that my students are poorly prepared [in mathematics]. If changing the content of the courses [in high school] or the methods of teaching or the teachers themselves would result in a more adequately prepared student, let us bring about the change by revolution if necessary....

126. ... As a matter of fact I believe that nearly one-third of our present college student body will not be benefited to any degree by any course in any college. For the student in this group it is immaterial what he takes or does in college. However, he will be happier by exposing himself to courses other than mathematics....

127. ... Again (may I refer you to your own study[4]), there are too many teachers [of mathematics] who do not teach the subject at all. They merely assign lessons and listen to recitations....

[4] A Critical Study of the Teaching of Elementary College Mathematics. Bureau of Publications. 1931.

128. ... Mathematics cannot be dispensed with by mankind. Why fuss?...

129. ... The formal, manipulative, exercise-working teacher of mathematics is in my opinion a much greater menance to mathematics than those intellectual dumps—the colleges of education....

130. ... The worst trouble, as I see it, is the poor quality of teaching that is generally done in high schools, preparatory schools, and other pre-college schools. And the thing that will ultimately do the most good will be the replacing of those older teachers by able, competent and interested teachers of Ph.D. rank. More Ph.D.'s should go into this secondary field....

131. ... Most students should take at least one course in mathematics in each of their college years. Exception: those specializing in such nonsense as English poetry, for which the ability to think may be a handicap....

132. ... Every pupil should be required to know the nature and uses of such major subjects as history, literature, mathematics, science, and social relations. After this "exposure" he should follow the lines to which he is reasonably adapted....

Mathematical Notes

Edited by
L. J. ADAMS

The spring meeting of the Maryland-District of Columbia-Virginia Section of the Mathematical Association of America was held at the United States Naval Academy, Annapolis, Maryland on Saturday, May 9, 1936. Professor G. T. Whyburn, of the University of Virginia, presided over both sessions. The following six papers were presented:

1. *An examination for instructors in mathematics.* Professor G. R. Clements, U. S. Naval Academy.
2. *On the abstract definition of a group.* Dr. Abraham Sinkov, Washington, D. C.
3. *An algebraic approach to weather forecasting.* Professor L. E. Root, U. S. Naval Academy.
4. *Newton's method and the method of iteration.* Professor J. M. Stetson, College of William and Mary.
5. *A method for finding equations of composite surfaces.* Professor G. C. Vedova, St. John's College.
6. *Some curious aspects of mathematical history.* Professor Tobias Dantzig, University of Maryland.

The fall meeting will be held at the Bureau of Standards, Washington, D. C. on Saturday, December 5, 1936. The chairman for the year 1936-37 will be Dr. John Williamson, who succeeds Professor G. T. Whyburn in that capacity. Dr. Williamson is on the faculty of The Johns Hopkins University.

The 333rd meeting of the American Mathematical Society was held jointly with the meeting of the Pacific Division of the American Association for the Advancement of Science at the University of Washington, Seattle on June 18, 1936. Professor H. P. Robertson of Princeton University delivered an address on *Geometry and physical space-time,* at the invitation of the Program Committee. A special anniversary program was sponsored by the Department of Mathematics of the University of Washington and the American Mathematical Society. The speakers at this special program were alumni of the University of Washinton, and included Professor E. T. Bell, of the California Institute of Technology, who spoke on *Highlights of mathe-*

matical biography, and Professor Harold Hotelling, of Columbia University, whose address was entitled *Correlated Vectors*. At the meeting of the Society some fifteen papers were read, while fourteen others were presented by title.

The Wisconsin Section of the Mathematical Association of America held its fourth annual meeting at the University of Wisconsin, Madison, on Saturday, May 9, 1936. Papers read were:

1. *The use of homogeneous coordinates in the teaching of elementary mathematics.* Professor H. H. Pettit, Marquette University.
2. *Graphical representation of the real and complex roots of cubic equations.* Mr. J. A. Ward, University of Wisconsin.
3. *Extension of the concept of length.* Professor R. L. Jeffery, Acadia University, Canada.
4. *Linear algebras.* Professor H. H. Conwell, Beloit College.
5. *Some applications of vector methods to differential geometry.* Professor G. A. Parkinson, University of Wisconsin, Extension Division.
6. *Mathematics placement tests.* Miss Elizabeth Knight, State Teachers College, Milwaukee.
7. *Some curriculum and evaluation problems related to mathematics.* Professor M. L. Hartung, Department of Educational Methods, University of Wisconsin.

The Beta Chapter of K. M. E. (honorary mathematics fraternity) of Mississippi State College publishes an annual bulletin entitled *The Magnolia*. This bulletin contains a resumé of the chapter activities for the year, and a roster of members and officers. In addition to social events the programs include addresses on mathematical and scientific topics. Professor C. D. Smith is sponsor of Beta Chapter. The K.M.E. fraternity was founded in 1932. It welcomes the membership of other mathematics clubs now functioning independently.

T. Hayashi, founder of the *Tôhoku Mathematical Journal* of Japan, died on October 4, 1935.

The Journal of the Royal Statistical Society of England contains a department devoted to listing statistical and economic articles in current periodicals. The offices of this journal are at N. 4, Portugal St. (Kingsway) W. C. 2, London.

Professor A. C. Dixon, F. R. S., eminent English mathematician died at the age of seventy years on May 4, 1936. Professor Dixon was president of the London Mathematical Society for the years 1931-1933.

The School of Education of the University of Michigan announces a course in the *Teaching of Mathematics* to be conducted via radio, for this semester. The course will be conducted by Dr. Raleigh Schorling, whose lectures will be supplemented by those of visiting speakers. The broadcasts will be heard over station WJR at 9 a. m. (E. S. T.) every Saturday for eighteen weeks. Regular credit will be given through the Extension Division of the university.

Sir James Jeans presented the Mathematical Association of England with a copy of the edition of *Euclid's Elements* printed at Basle in 1533. The book was formerly the property of the Royal Astronomical Society.

A complete list of abstracts of all mathematical research papers published in Japan during the scholastic year is contained in Vol. XII, No. 4 of the *Japanese Journal of Mathematics*. The index to the list shows a total of 234 papers. There are listed at the beginning of the abstracts the fifteen Japanese journals publishing research papers in mathematics.

On May 14, 1936 Dr. A.C. Aitken delivered an address on *Arithmetical Recreations* before the meeting of the London Mathematical Society. The meeting was held at the home of the Royal Astronomical Society, Burlington House.

Rice Institute, of Houston, Texas, publishes a pamphlet containing important papers of the members of the faculty and those of visiting professors. The January, 1936 number was devoted to *Fundamental Theorems Concerning Point Sets*, by Professor R. L. Moore, of the University of Texas.

The American Mathematical Society announces meetings as follows:

New York City. October 31, 1936.

Lawrence, Kansas. November 27, 28, 1936.

Los Angeles, California. November 28, 1936.

Duke University, Durham, North Carolina. Annual meeting. December 29, 1936-January 1, 1937.

The third edition of Professor R. C. Archibald's *Outline of the History of Mathematics* is now available. The price is fifty cents per copy, postpaid. Copies may be secured through Professor W. D. Cairns, 33 Peters Hall, Oberlin, Ohio. The third edition contains new material, including the recent finds in the history of Babylonian mathematics.

The Mathematical Association of America operates a placement bureau for men and women holding a Ph.D. in mathematics. The activities of the bureau are directed by Professor E. J. Moulton, Northwestern University, Evanston, Illinois. In addition to placement, the bureau helps to arrange exchange professorships.

Harvard University, in connection with the celebration of its Tercentenary, was host to the following organizations during the week of August 31-September 5, 1936: American Mathematical Society, Mathematical Association of America, Institute of Statistics, Association for Symbolic and the American Astronomical Society. Addresses by invitation included:

1. *Uncertain inference.* Professor R. A. Fisher, University of London.
2. *The Indian mathematician, Ramanujan.* Professor G. H. Hardy, University of Cambridge.
3. *Truth in mathematics and logic.* Professor Rudolph Carnap, Deutsche Universität, Prague.
4. *L'extension du calcul tensoriel aux géométries non affine.* Professor E. J. Cartan, University of Paris.
5. *Waring's problem and its generalizations.* Professor L. E. Dickson, University of Chicago.
6. *The relativistic problem of several bodies.* Professor Tullio Levi-Civita, University of Rome.
7. *The cosmical constant and the recession of the nebulae.* Professor A. S. Eddington, University of Cambridge.
8. *Topics in general analysis.* Professor E. W. Chittenden, University of Iowa. (Colloquium Lectures).
9. *Methods of modern analysis in potential theory.* Professor G. C. Evans, University of California.

A total of 67 papers were presented before the meetings of the American Mathematical Society, while 46 others were presented by title only.

Social functions and excursions for the visitors were arranged by Harvard University.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 100. Proposed by *W. V. Parker*, Georgia Tech.

If P_1, P_2, P_3, P_4 are the incenter and the three excenters of a triangle, the system of conics on them is a system of equilateral hyperbolas with centers on the circumscribing circle of the triangle.

A solution for this problem has already been published. The following discussion is an outgrowth of the problem, however, and it is submitted by ***Robert C. Yates***, University of Maryland, under the title, ***A Net of Hyperbolas.***

The general equation of the conic section represents a rectangular hyperbola if the sum of the coefficients of the square terms is zero. Let:

$$(1) \qquad x^2+2axy-y^2+2bx+2cy+d=0$$

represent all rectangular hyperbolas in the plane. Any one is determined by four points unless they form an orthocentric group. The first partial derivatives are:

$$(2) \qquad \left.\begin{array}{r} x+ay+b=0 \\ ax-y+c=0 \end{array}\right\}$$

which give, as their solution, the center of the hyperbola. We have, accordingly:

$$(3)\qquad \left.\begin{aligned}(1+a^2)x+b+ac&=0\\(1+a^2)y+ab-c&=0\end{aligned}\right\}$$

Let us suppose the quantities b and c determined—thereby fixing a net (or ∞^2) of these hyperbolas. Equations (3), then, depend on a alone and constitute the parametric equations of the locus of the center. For the elimination, let $a=\tan\Theta$. Then:

$$\left.\begin{aligned}x&=-(b\cdot\cos^2\Theta+c\cdot\sin\Theta\cos\Theta)=-(b/2+b\cdot\cos2\Theta/2+c\cdot\sin2\Theta/2)\\y&=-(b\cdot\sin\Theta\cos\Theta-c\cdot\cos^2\Theta)=-(b\cdot\sin2\Theta/2-c/2-c\cdot\cos2\Theta/2)\end{aligned}\right\}$$

or

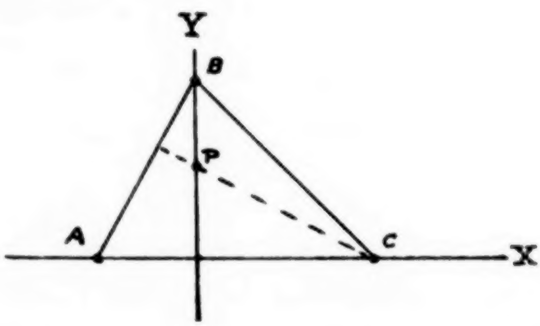
$$\left.\begin{aligned}2x+b&=-(b\cdot\cos2\Theta+c\cdot\sin2\Theta\\2y-c&=-(b\cdot\sin2\Theta-c\cdot\cos2\Theta)\end{aligned}\right\}$$

Squaring and adding, we find:

$$(4)\qquad \boxed{x^2+y^2+bx-cy=0}$$

a circle through the origin.

One hyperbola of the system (1) is $xy=0$ (the axes of reference, given by $a=\infty$). Since all rectangular hyperbolas on three points, A, B, C, cut out with each other a fourth, P, which completes an orthocentric group, the points in which each curve cuts the axes are orthocentric. These are given by $x^2+2bx+d=0$ and $y^2-2cy-d=0$. Since (4) contains the origin, which is the foot of an altitude, we may select the other two sides and corresponding altitudes as axes to exhibit the same property. Thus (4) is the Nine-point circle of A, B, C, or the circumcircle of the pedal triangle.



No. 109. Proposed by *Wm. E. Byrne.*

Evaluate $$\int_b^a \frac{1}{y^2}\left[\frac{\sqrt{2a^2-y^2}}{a\sqrt{2}}-\frac{a}{\sqrt{a^2+y^2}}\right]dy$$

Solution by *The Proposer.*

An element of the integral near $y=0$ is

$$\frac{1}{y^2}\left\{1-\frac{y^2}{4a^2}+\ldots.-\left[1-\frac{1}{2}\frac{y^2}{a^2}+\ldots.\right]\right\}=\frac{1}{4a^2}+\ldots.$$

hence finite

Now $$I=\int\frac{\sqrt{2a^2-y^2}}{a\sqrt{2}y^2}\,dy=\int\frac{a\sqrt{2}\cos\theta\cdot a\sqrt{2}\cos\theta d\theta}{a\sqrt{2}\cdot 2a^2\sin^2\theta}$$

Where $y=a\sqrt{2}\sin\theta$

$$\theta=\text{Arcsin}\frac{y}{a\sqrt{2}}=\frac{1}{a\sqrt{2}}\int\cot^2\theta d\theta$$

$$=-\frac{1}{a\sqrt{2}}\cot\theta-\frac{\theta}{a\sqrt{2}}$$

$$=-\frac{1}{a\sqrt{2}}\;\frac{\sqrt{2a^2-y^2}}{y}-\frac{1}{a\sqrt{2}}\arcsin\frac{y}{a\sqrt{2}}$$

$$J=\int^a\frac{dy}{y^2\sqrt{a^2+y^2}}=\int\frac{a\cdot a\sec^2\theta d\theta}{a^2\tan^2\theta\cdot a\sec\theta}$$

$$=\frac{1}{a}\int\frac{\cos\theta d\theta}{\sin^2\theta}=-\frac{1}{a\sin\theta}=-\frac{\sqrt{a^2+y^2}}{ay}$$

where $y=a\tan\theta,\quad \theta=\arctan y/a$

It should be noticed here that the θ's of the I and J integrals are not the same.

These results taken together are

$$\int \frac{1}{y^2}\left\{\frac{\sqrt{2a^2-y^2}}{a\sqrt{2}}-\frac{a}{\sqrt{a^2+y^2}}\right\}dy$$

$$=-\frac{1}{a\sqrt{2}}\;\frac{\sqrt{2a^2-y^2}}{y}-\frac{1}{a\sqrt{2}}\arcsin\frac{y}{a\sqrt{2}}+\frac{1}{a}\;\frac{\sqrt{a^2+y^2}}{y}=F(y)$$

and

$$\int_0^a \frac{1}{y^2}\left\{\frac{\sqrt{2a^2-y^2}}{a\sqrt{2}}-\frac{a}{\sqrt{a^2+y^2}}\right\}dy=\lim_{\substack{y\to 0\\(y>0)}}[F(a)-F(y)]$$

$$F(a)=-\frac{1}{a\sqrt{2}}\cdot\frac{a}{a}-\frac{1}{a\sqrt{2}}\arcsin\frac{1}{\sqrt{2}}=-\frac{1}{a\sqrt{2}}-\frac{\pi}{4a\sqrt{2}}$$

$$\lim_{y\to 0}F(y)=\frac{1}{a}\lim_{y\to 0}\left[\frac{\sqrt{a^2+y^2}}{y}-\frac{1}{\sqrt{2}}\;\frac{\sqrt{2a^2-y^2}}{y}\right]=0$$

Hence our original integral equals

$$-\frac{1}{a\sqrt{2}}\left[1+\frac{\pi}{4}\right]$$

No. 113. Proposed by *W. Van Parker*, Georgia School of Technology.

If K is any given positive integer, show that the equation

$$(x+y)^2+(x-y)=2K$$

has a unique solution in integers such that

$$x\geq 0,\ y>0$$

Solution by *T. A. Bickerstaff.*

$$4(x+y)^2+4x-4y=8K$$

$$4(x+y)^2+4(x+y)=8(K+y)$$

$$4(x+y)^2+4(x+y)+1=8(K+y)+1$$

$$2(x+y)+1=\sqrt{8(K+y)+1}$$

Now since $8(K+y)+1$ is a square,

$$K+y=\frac{n(n+1)}{2} \tag{1}$$

where n is a positive integer; and

$$x+y=n \tag{2}$$

Hence we have

$$y=\frac{n(n+1)}{2}-K>0 \tag{3}$$

and

$$x=K-\frac{n(n-1)}{2}\geq 0 \tag{4}$$

From which

$$(n+1)n>2K\geq n(n-1) \tag{5}$$

Now for every K, there is one n exactly, and therefore one solution (3), and (4) for x and y.

Also solved by *A. C. Briggs, W. B. Clarke*, and the *Proposer*.

No. 118. Proposed by *W. V. Parker*.

If $f(x)$ is a polynomial of degree $2n+1$, and $g(x)$ is a polynomial of degree $2n$ determined by the conditions

$$f\left[\frac{(2n-k)a+kb}{2n}\right]=g\left[\frac{(2n-k)a+kb}{2n}\right] \quad (k=0,\ 1,\ldots .2n),$$

then
$$\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha}\frac{\lambda f(x)+\mu g(x)}{\lambda+\mu}dx=\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha}f(x)dx \qquad (\lambda+\mu\neq 0).$$

Solution by the *Proposer*.

Let
$$p(x)=\frac{\lambda f(x)+\mu g(x)}{\lambda+\mu}$$

and make the substitution

$$x=y+\frac{a+b}{2}$$

so that $p(x)=P(y)$ and $g(x)=G(y)$. Write $H(y)=P(y)-G(y)$, then

$$H\left[\frac{n-k}{2n}(a-b)\right]=0,\ (k=0,1,\ldots.2n).$$

$H(y)$ is, therefore, an odd function of y and hence

$$\int_{-\alpha}^{\alpha} H(y)dy=0 \text{ or } \int_{-\alpha}^{\alpha} P(y)dy=\int_{-\alpha}^{\alpha} G(y)dy.$$

If now we make the substitution $y=x-\dfrac{a+b}{2}$, we have

$$\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha}\frac{\lambda f(x)+\mu g(x)}{\lambda+\mu}dx=\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha} g(x)dx$$

In particular if $\lambda=1$ and $\mu=0$, $p(x)=f(x)$

and hence
$$\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha}\frac{\lambda f(x)+\mu g(x)}{\lambda+\mu}dx=\int_{\frac{a+b}{2}-\alpha}^{\frac{a+b}{2}+\alpha} f(x)dx$$

No. 119. Proposed by *H. T. R. Aude*, Colgate.

If a triangle is placed so that its circumcenter is at the origin and if the coordinates of the vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3), the orthocenter is located at the point $(x_1+x_2+x_3, y_1+y_2,+y_3)$.

Solution by *C. A. Balof.*

Since the median point trisects each median, its coordinates are $\{{}^1/_3(x_1+x_2+x_3), {}^1/_3(y_1+y_2+y_3)\}$. But the median point also trisects the line from the circumcenter to the orthocenter. Hence, using the

equations for a point of division, the coordinates of the orthocenter are $(x_1+x_2+x_3,\ y_1+y_2+y_3)$.

Also solved by *A. C. Briggs*, *Karleton W. Crain*, and *Brother Northbert*.

No. 120. Proposed by *H. T. R. Aude*, Colgate.

The usual interpretation of the radical symbol is that the sign in front of the radical, either written or understood, is the only one taken. It is, therefore, a problem to find the value of K for which the equation

$$\sqrt{X^2-9}+K-X=0$$

has a solution.

Solution. Solving the equation for X, we have

$$X=\frac{K^2+9}{2K}$$

Substituting in the original equation, we obtain

$$\sqrt{\frac{K^4-18K^2+81}{4K^2}}+K-\frac{K^2+9}{2K}=0$$

This last equation is true only when the radical is taken as

$$\frac{9-K^2}{2K}$$

which must be positive. Therefore the original equation will have a solution if $K<-3$ or if $0<K<3$.

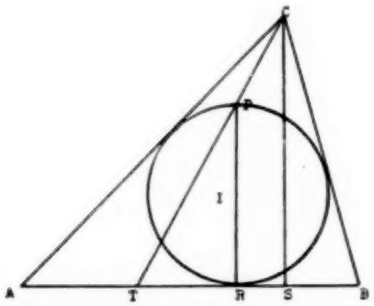
Also solved by *A. C. Briggs*.

No. 121. Proposed by *Walter B. Clarke*.

Show that any verbisector (cevian from any vertex to the point halfway around the perimeter of a triangle), the incircle and its diameter perpendicular to the side cut by the verbisector are concurrent.

Solution by *Karleton W. Crain*, Purdue University.

In the triangle ABC draw the altitude CS, and the diameter RP of the incircle parallel to CS. Draw the lines CP and PT, where T is the point on AB which is halfway around the perimeter of the triangle



from C. Now if triangle CST and PRT can be proved similar, then C, P, and T will lie on a straight line and the truth of the problem be established. In other words, we wish to verify the following identity,

$$SC/TS = RP/TR.$$

From the figure, $\qquad SC = a \sin B,\ RP = 2r,$

$$TS = s-a-SB = s-a-a\cos B,\ TR = s-a-RB = s-a-r\cot B/2$$

where a, b, c, are the sides of the triangle and $2s = a+b+c$, and r is the radius of the incircle.

Using formulae from Trigonometry,

$$SC = 2a\sin B/2\cos B/2 = 2a\sqrt{\frac{(s-c)(s-a)}{ca}}\cdot\sqrt{\frac{s(s-b)}{ca}}$$

$$RP = 2\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$TS = s-a-a(c^2+a^2-b^2)/2ca.$$

$$TR = s-a-\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}\cdot\sqrt{\frac{s(s-b)}{(s-c)(s-a)}}.$$

Substituting these values into the proposed identity, both sides reduce to

$$2\frac{\sqrt{s(s-a)(s-b)(s-c)}}{s(b-a)}.$$

Also solved by *C. A. Balof*, *A. C. Briggs*, and *Henry Schroeder*.

No. 122. Proposed by *R. A. Miller.*

Given the circle O with diameters AB and CD perpendicular. $\overset{\frown}{AQ}=\overset{\frown}{AS}=\overset{\frown}{CM}=\overset{\frown}{CR}=60°$, $AT=CV={}^2/_3AO$. Draw O' through MVR, and O'' through QTS. Let P be the point of intersection of O' and O'' which lies in O. Prove or disprove that $PAO=30°$.

Solution by *C. A. Balof,* Lincoln College (Ill.)

Taking the center of circle O at the origin, the coordinates of A being $(-r, O)$ and of C being (O, r), the equations of O' and O'' are

$$O' : 9x^2+9y^2-48ry+15r^2=0$$

$$O'' : 9x^2+9y^2+48rx+15r^2=0$$

The coordinates of P are then $[r(-8+\sqrt{34})/6,\ r(8-\sqrt{34})/6]$. Thus $\tan PAO=(8-\sqrt{34})/(-2+\sqrt{34})$ and PAO is not equal to $30°$.

Also solved by *A. C. Briggs, W. B. Clarke,* and *The Proposer.*

No. 123. Proposed by Richard A. Miller, State University of Iowa.

Determine the relations which exist between the angles of a triangle such that:

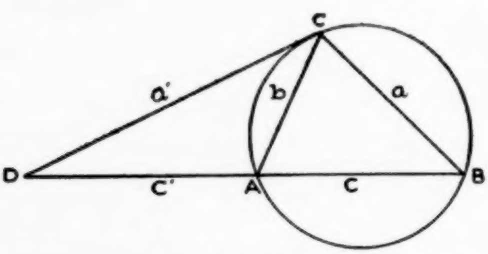
(a) $a^2=b(b+c)$

(b) $bc^2=(a-b)^2(a+b)$

(c) $a^2bc^3=[(a^2-b^2)^2-b^2c^2]\,[(a^2-b^2)-bc]$

where $a, b, c,$ are lengths of sides.

Solution by *Proposer.*



(a) Let ABC be the triangle with the tangent to circumcircle at C cutting AB at D. Denote DC, DA by a', c'.

$\angle DCA=\angle ABC$ Hence $\triangle DCA\sim\triangle DCB$ and

(1) $$a=\frac{ba'}{c'}$$

Substitution of (1) in (a) gives

(2) $$a'^2=\frac{c'^2(b+c)}{b}$$

But $$a'^2=c'(c'+c)$$

(3) hence $$c'=b$$

$\angle D=\angle ACD=\angle ABC=\angle B$ where $\angle D$ is measured by $^1/_2(\widehat{BC}-\widehat{AC})$

$\angle D=\angle B={}^1/_2(2A-2B)$ from which $A=2B$

(b) To show this, substitute (1) in (b), which gives upon reduction:

$$c^2=\frac{b^2(a'-c')^2(a'+c')}{c'^3}=\frac{b^2(a'-c')(a'^2-c'^2}{c'^3}$$

which reduces to, by aid of (3),

$$b^2=\frac{cc'^2}{a'-c'}=c'(a'+c').$$

This last relation shows by means of (a) that $\angle D\leqq\angle 2ACD=2B$. Hence $A=3B$.

(c) Using the relations (1) and (3) as before in (c) and finally (b), we have $D=3B$. Hence $A=4B$

Note: In the first printing, (c) appeared

$$a^2bc^3=[(a^2-b^2)^2-b^2c^2]\,[(a^2-b^2)^2-bc.]$$

This gives $A=B$.

Also solved by *C. A. Balof* and *A. C. Briggs.*

No. 124. Proposed by *Nathan Altshiller-Court,* University of Oklahoma.

The point of contact of each face of a tetrahedron with the escribed sphere relative to this face* is joined to the mid-point of the altitude

*Nathan Altshiller-Court, *Modern Pure Solid Geometry,* p. 76. The Macmillan Company, 1935.

relative to the face considered. The four lines thus obtained are concurrent.

Solution by *The Proposer.*

Let (I) be the inscribed sphere of the tetrahedron $A\,B\,C\,D$ and (I_a) the escribed sphere x relative to the face $B\,C\,D$, I, I_a the centers of these spheres, and P, Q their points of contact with the plane $B\,C\,D$.

The point A is a center of similitude of the two spheres, and the two radii IP, I_aQ are parallel, hence the line $A\,Q$ meets IP in the diametric opposite R of P on (I).

Let $(A\,H)$ be the sphere having for diameter the altitude $A\,H$ of the tetrahedron. The points A, R are directly homologous on the spheres (I), $(A\,H)$, hence the external center of similitude X of these spheres lies on the line AR. On the other hand $B\,C\,D$ is an external common tangent plane of the two spheres (I), $(A\,H)$, hence this plane passes through X, i. e., the point X coincides with Q.

Now the center of similitude Q of (I) and $(A\,H)$ lies on the line of centers of these two spheres, hence the line QM joining Q to the mid-point M of $A\,H$ passes through the incenter I of the tetrahedron. Similarly for the other faces of the tetrahedron. Hence the proposition.

Note. A similar problem in the plane was considered in the Bulletin de sciences mathématiques et physiques élémentaires, Vol. 9 (1903-1904), p. 312, Q. 1451.

No. 125. Proposed by *Nathan Altshiller-Court,* University of Oklahoma.

If an arbitrary plane through the centroid G of a tetrahedron $DABC$† cuts the edges DA, DB, DC in the points P, Q, R, we have, both in magnitude and in sign,

$$\frac{AP}{PD}+\frac{BQ}{QD}+\frac{CR}{RD}=1$$

Solution by *The Proposer.*

Let the lines PG, QG, RG meet the lines QR, RP, PQ in the points U, V, W, respectively. The median AG of the tetrahedron meets the face DBC in the centroid G_a of the triangle DBC, and this point lies on the line DU, for the two lines lie in the same plane GAD. Applying

†Nathan Altshiller-Court. *Modern Pure Solid Geometry,* p. 52, Art. 171. The Macmillan Company, 1935.

Menelaus' theorem to the triangle APQ and the transversal DUG_a, we have

$$\frac{PD}{DA}+\frac{AG_a}{G_aG}+\frac{GU}{UP}=-1$$

hence $$4(PD:DA)=UP:GU,$$

or $$AD:PD=4(GU:PU)$$

Similarly

$$BD:QD=4(GV:QV),\ CD:RD=4(GW:RW).$$

Adding we obtain

$$\frac{AD}{PD}+\frac{BD}{QD}+\frac{CD}{RD}=4\left(\frac{GU}{PU}+\frac{GV}{QV}+\frac{GW}{RW}\right)=4‡$$

hence

$$\frac{AP}{PD}+\frac{BQ}{QD}+\frac{CR}{RD}=1$$

Note. The corresponding problem in the plane was considered in the Bulletin de sciences mathématiques et physiques élémentaires, Vol. 2 (1896-1897), p. 6, Q. 116.

Late solution, No. 116, by *C. A. Balof.*

PROBLEMS FOR SOLUTION

No. 130. Proposed by *H. T. R. Aude*, Colgate University.

A man states that he was x years old in the year x^2. He adds, if to the number of my years you add the number of my month, it equals the square of the date. When was he born?

No. 131. Proposed by *T. A. Bickerstaff*, University of Mississippi.

In the right triangle ABC, $(C=90°)$, equal line segments are drawn from A and B to A' and B' on the opposite sides, intersecting at O. Let the bisector of AOB' and $A'OB$ intersect $A'B$ and AB' at M and N respectively. Show that

$$CM\cdot A'B=CN\cdot AB'$$

‡Nathan Altshiller-Court, *College Geometry*, p. 131, Art. 244. Richmond, Va., 1925.

No. 132. Proposed by *William E. Byrne*, V. M. I.

Let (x_0, y_0) be a point P_0 of the curve C, $y=f(x)$. Under the assumptions that $f(x)$ admits a Taylor development about the point P_0 and that $f''(x_0), f'''(x_0) \neq 0$, find

$$\lim_{x_1 \to x_0} \frac{x_1^n + x_2^n}{x_1 + x_2}, \quad (n=2, 3, \ldots.),$$

where $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ are points of C and the chord P_1P_2 is parallel to the tangent P_0T_0 at P_0. What happens if $f''(x_0)=0$, $f'''(x_0) \neq 0$?

Book Reviews

Edited by

P. K. SMITH

Anekdoten aus dem Leben deutscher Mathematiker. By Johannes Mahrenholz. B. G. Teubner, Berlin and Leipzig, 1936. IV+44 pp. and frontispiece, 8° in paper covers. Price 1.20 mark.

The author, who is a mathematics teacher in the Adolf Hitler high school in Cottbus, writes in the preface that he expects this little monograph will find many critics. Probably he had in mind the serious question as to whether a collection of anecdotes constitutes mathematical history. Mere anecdotes in themselves certainly are not the history of mathematics, but they usually do no harm, and often throw interesting or valuable sidelights on it, and vitalize it.

We find the following names: Adam Riese, Stifel, Kepler, Erhard, Weigel, Leibniz, Euler, Lambert, Kästner, Gauss, Abel, Steiner, Weierstrass, and Karl Heinrich Schellbach. What Mahrenholz has done is to collect miscellaneous scraps of personal information about these men, primarily from printed sources which however are often rather inaccessible. The stories touch in part on their scientific work, and are in general somewhat humorous in tone. One should not expect too much from such a small collection; it is intended more for relaxation than reference, and is to be used by amateurs and those teaching in grade or high school. Its value lies in the possible stimulus to those who read it, that they may later study the printed biographies and the author's other sources, with a view to exhaustive investigation. Format and typography are good.

University of Illinois. G. WALDO DUNNINGTON.

Mathematics of Modern Engineering. Volume I. Doherty and Keller, John Wiley and Sons, New York, 1936. 314+xv pages.

This book is the first of two volumes based on the Advanced Engineering Course of the General Electric Company. It is divided into four chapters as follows: Mathematical Formulation of Engineering Problems, Basic Engineering Problems, Vector Analysis Heaviside's Operational Calculus.

Chapter I deals with the processes involved in passing from a given engineering problem stated in words to the mathematical set-up of the equations to be solved after due attention has been paid to simplifying assumptions, plan of attack, choice of coordinate systems and units of measurement. The following chapters afford many examples of this procedure.

Chapter II is devoted to ordinary differential equations with constant coefficients with applications to electrical circuit and mechanical problems; determinants with applications to the operational method of determining particular solutions of a system of linear differential equations; Fourier series (brief); solutions of higher degree and transcendental equations with accent on Graeffe's root-squaring method; dimensional analysis; graphical and numerical methods of solving differential equations.

Chapter III takes up the essentials of vector analysis, with applications to the derivation of some of the partial differential equations of mathematical physics, vector magnetic theory and dyadics.

In Chapter IV appear a brief summary of the theory of functions of a complex variable and a discussion of the Heaviside "indicial admittance" with Bromwich's solution accompanied by manipulative devices and applications of the operational calculus.

There are misprints or omissions easily found by the reader on pages 37, 42, 64, 83, 89, 99, 103, 134, 192, 202, 263, 264, 265. On page 41 the tacit assumption is made that $F(a)$ in (89), $F(-a^2)$ in (90), (91), $f(-a^2)$ in (92), (93) are not zero. In several of the answers a similar assumption is made. The answer given for 9(*b*), page 43, for

$$dy/dx = \sqrt{\frac{1-y}{1-x}}$$

indicates it was assumed that $y<1$, $x<1$, whereas it is also possible that $y>1$, $x>1$. On page 48 (as well as on page 36) operating on both members of a differential equation by a polynomial in $p(=d/dt)$ is spoken of as "multiplying" by the operator. On page 67

$$\frac{1}{p}f(t) = \int_0^t f(t)dt$$

does not meet the requirements of (72), page 35, since

$$p\left[\frac{1}{p}f(t)\right] = p\int_0^t f(t)dt = f(t)$$

and

$$\frac{1}{p}[pf(t)] = \int_0^t f'(t)dt = f(t) - f(0)$$

On page 194 the values of θ used as limits of integration do not check with the indications of the figure. On page 265 q_s was defined as

$$\int_0^t i_s dt$$

and hence $q_s(0) = 0$.

This text is unusual in that it combines the viewpoint of the mathematician with that of an industrial engineer. It does not avoid the heavy numerical work that appears in applied problems. It should prove quite thought-provoking both to the engineering student and the instructor of mathematics; in fact, every mathematician who is teaching engineering students would profit also from reading the foreword to instructors. The authors have insisted that the book is not a mathematics text but rather a guide showing the scientific route for the engineer to take from physics to mathematics. Exception might be taken to the following statements which occur on page 7, Chapter II; "We may distinguish between the engineering and the mathematical points of view. The study of ordinary differential equations, from a mathematical point of view, consists of three parts:

(a) A proof of the existence of a solution of a single equation or system of equations.

(b) The investigation of the properties of the solution: continuity, differentiability, analyticity, and integrability of the solution with respect to both the independent variable and important parameters present.

(c) The construction of a solution in a form suitable for the use at hand.

The study of differential equations from an engineering point of view is different. It consists mainly of but two parts:

(a) The derivation of the differential equations.

(b) The solution of the differential equation."

Both parts (a) and (b) of the engineering point of view enter into the mathematical point of view: the mathematician merely goes into the

problem more thoroughly than does the engineer. This book merits the attention of physicists, mathematicians and engineers; the second volume will be awaited with interest.

Virginia Military Institute. WILLIAM E. BYRNE.

Plane and Spherical Trigonometry. By Alfred L. Nelson and Karl W. Folley. Harper and Brothers, New York, 1936, ix+186 pages.

In the reviewer's opinion, one of the most valuable features of this book is the fact that the analytic aspects of trigonometry are introduced early in the book and are presented, with little interruption, until the subject is completely covered. Radian measure is introduced in the first chapter, and throughout the book angles are measured in radians as well as in the units of the sexigesmal system.

In Chapter I the general definitions of the trigonometric functions are presented and their properties and general inter-relations are developed by logical and easy steps. The chapter is concluded with a discussion of trigonometric identities.

The development of the analytic properties of the trigonometric functions is interrupted by Chapter II, which deals with the functions of an acute angle and their applications to the solution of the right triangle, to vectors, and to projections. The material in this chapter serves well to clarify the student's conception of the meaning of the functions and to give him an appreciation of their practical value.

The authors resume their analytic sequence in Chapter III with a treatment of reduction formulas and graphs, and the next three chapters deal with the trigonometric functions of two angles, inverse trigonometric functions, and trigonometric relations, respectively. Reduction formulas for

$$\pi \pm \theta, \qquad 2\pi \pm \frac{\pi}{2} \pm \theta, \qquad \frac{3\pi}{2} \pm \theta$$

are presented, but no general reduction formula is given. The authors have succeeded in obtaining the formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ without departing from the use of the general definitions of the trigonometric functions, but in doing so they have resorted to the idea of a change of coordinate axes.

Chapter VI deals with logarithms, and Chapter VIII with the logarithmic solution of triangles. The portion of the book dealing with plane trigonometry is concluded with a chapter treating com-

plex numbers. Spherical trigonometry is covered briefly but clearly in twenty pages.

The book is clearly written and the theory is illustrated with many completely solved examples. The figures are large and are carefully drawn. There is an abundance of well chosen exercises. Five place tables of logarithms, logarithmic functions and natural functions compiled by Lennes and Merrill are bound in the book. It is the opinion of the reviewer that the book is competent, well prepared, and that it will be very popular.

Texas Technological College. FRED W. SPARKS.

Algebra for College Students. By H. R. Willard and N. R. Bryan, New York, Scott, Foreman and Company, 1936. viii+386 pages. $2.00.

The scope of the text permits a long or short course. The subject matter covered and continuity in development are approximately the same as in other standard works, except that Interest and Annuities do not immediately follow Logarithms or Progressions. The proverbial chapter on Complex Numbers does not precede Theory of Equations.

In the second chapter, logarithms are introduced in conjunction with the laws of exponents, thus giving the students as the preface states, "a handy tool to use in computation in later work both in algebra and in other fields."

In the extensive review of pre-college mathematics some advanced material has been incorporated. An abundance of illustrative examples of types varying from payment for work to physics "that will interest the student and will furnish practice for developing skill in manipulation" of the processes under consideration have been included. Contrary to many recent publications, answers are given to all problems. There is a conspicuous absence of foot notes, so rarely read by the average student.

The graphical method is used extensively in clarifying theory and checking problems. The notion of derivatives is introduced in the chapter on Theory of Equations, but Newton's method for the approximate solution of an equation has been omitted. Horner's method is, of course, given. Nearly eleven pages are devoted to the solution of the general cubic and quartic equations.

As the name implies, the authors in their ten years of development of the material in their text, have had the student's viewpoint uppermost in mind. Brief introductory paragraphs are given before new topics are taken up, giving the reader the meat of the subject in a few words. The book appears to be thoroughly teachable.

Agnes Scott College. HENRY A. ROBINSON.